

STRESS ANALYSIS IN HELICAL SPRINGS WITH CLOSED END COILS MACHINED FROM CYLINDRICAL SLEEVES

SUMMARY

The review of stress calculation methods in helical springs with rectangular cross-section wire is introduced in this paper. It is demonstrated that methods described in literature have various accuracy. Also, the modern construction of helical spring machined from tubular blanks is presented. It is indicated, using Finite Elements Method, that existing for classical springs with opened ends stress calculating methods, are not suitable for springs with capped ends, because the stresses calculated from them are much smaller than the real ones.

Keywords: helical springs, FEM

ANALIZA NAPRĘŻEŃ W SPRĘŻYNACH ŚRUBOWYCH O ZAMKNIĘTYCH ZWOJACH KOŃCOWYCH, WYCINANYCH Z TULEI CYLINDRYCZNYCH

W pracy zaprezentowano przegląd metod obliczania naprężeń w sprężynach śrubowych wykonanych z drutu o przekroju prostokątnym. Wykazano, iż metody opisane w literaturze mają zróżnicowaną dokładność. Przedstawiono nowoczesną konstrukcję sprężyny śrubowej o zamkniętych zwojach końcowych, wycinanej laserem z tulei cylindrycznej. Przy użyciu metody elementów skończonych wykazano, że istniejące metody obliczeniowe dla typowych sprężyn śrubowych zastosowane do obliczania sprężyn o zamkniętych zwojach końcowych dają wyniki obarczone znaczącym błędem.

Słowa kluczowe: sprężyny śrubowe, MES

1. INTRODUCTION

Helical springs are wide-spread elastic elements and are very often used in machines construction. Usually helical springs are produced by coiling steel wire of circular or rectangular section. Wires with small diameter ($d \leq 8\div 10$ mm) are mostly cold-coiled after heat treatment. After forming springs are usually tempered in order to reduce residual stresses. Springs with bigger sections as well as springs with liable applications are hot-coiled and subsequently heat treated in order to increase mechanical properties.

Because of different applications and followed by them different requirements related to characteristic, ratio between maximum load and transversal dimensions, mounting solution etc. many different constructions of helical springs are used. The construction that is relatively new among others is spring machined from cylindrical sleeve. Springs of such type made with high precision without introducing re-

sidual stresses distinguish themselves with high accuracy of characteristic and possibility of various mounting ways with co-operating elements as it is shown in Figure 1.

The ways of spring mounting shown above enable their application in many-directions loaded systems. Such springs can therefore work both for stretching and compressing.

Thanks to mentioned high accuracy of characteristic, these springs can be applied in systems where because of certain reasons very small tolerance of positioning is demanded like e.g. in elastic seat of deflecting segments in slide thrust bearings of high overall dimensions.

Important advantage of such springs is also possibility of gaining very high stiffness, not possible in case of springs coiled from wire.

Besides the advantages presented above, helical springs similar to the ones shown at Figure 1 have also some disadvantages. The main disadvantage of such springs is high (comparing to helical spring coiled from wire with circular section, having the same stiffness and loaded with the same force) level of stresses during work. Additionally there is a stress concentration in the neighbourhood of place where the coils begin.

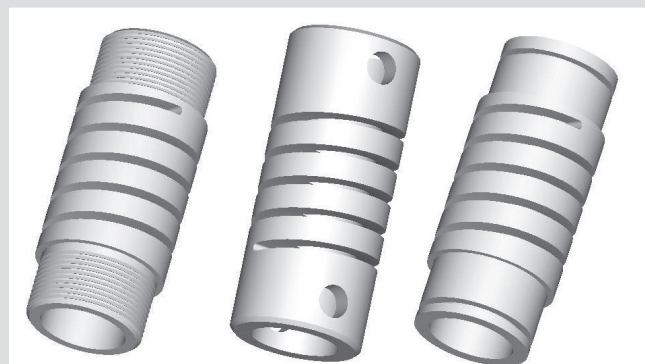


Fig. 1. Various ways of machined helical springs mounting

2. STRESS CALCULATION METHODS

There are some references in literature where authors present stress calculation methods. These methods are generally based on assumption that only torsional stresses influence the deformation, that spring is loaded ideally symmetrically and can twist around its axis although, the mounting conditions do not assure it in reality. In a latest

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publications [4, 5] the authors analyse the state of stress in springs based only on the Finite Elements Method.

In [1] authors give the following formula for shear stresses in spring coiled from wire with rectangular section

$$\tau = \frac{PD}{2W_0 \cdot g_k} \leq g_w \cdot k \quad (1)$$

where:

- P – the axial force loading the spring;
- $W_0 = \eta ba^2$ – stiffness of spring wire;
- η – coefficient depending on the ratio b/a ;
- g_k – the shape of section coefficient, depending on the ratio b/a and D/b read from diagram, where: b – shorter side of wire section, a – longer side of wire section;
- D – nominal spring diameter;
- g_w – dimension of section coefficient, taking into account difference between thickness of test bar and thickness of wire the spring is made of, read from diagram;
- k – allowable torsional stresses.

Żukowski in [2] presents formula (2), based on the Saint-Venant theory supplemented with Wahl's correcting coefficient W

$$\tau = \frac{PD}{2\mu ab^2} \cdot K \quad (2)$$

where:

- μ – depends on ratio the b/a ,
- K – Wahl's coefficient, given by formula

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C},$$

coefficient $C = D/b$.

Apart from giving formula (2) Żukowski gives approximating formula for maximum stresses in coiled helical spring made of rectangular section wire

$$\tau = \psi \frac{PD}{ab\sqrt{ab}} \quad (3)$$

where coefficient ψ is given by diagram as a function of ratios D/b and a/b .

Different formula for stresses in considered case gives E.I. Rivin in [3]

$$\tau = \frac{PD}{2\chi a^2 b} \quad (4)$$

Values of the coefficient χ are given in a table depending upon ratio b/a .

It has to be remarked that while formulas (1)–(3) concern coiled helical springs made of rectangular section wire,

the formula (4) is given in the way suggesting that it can be applied to springs with closed ends machined from cylindrical sleeves.

The values of shear stresses calculated on formulas (1)–(4) are presented in the Table 1. The spring parameters are as follows: nominal spring diameter $D = 35$ mm, coil section $a \times b = 7 \times 15$ mm², pitch $p = 10$ mm, material properties: steel 60S2, $k = 400$ MPa, Young's modulus $E = 206\,000$ MPa, Poisson's ratio $\nu = 0,3$. The value of loading force $P = 3074$ N was calculated according to formula (3) after substituting value of allowable torsional stresses in place of τ .

Table 1

P [N]	τ (1) [MPa]	τ (2) [MPa]	τ (3) [MPa]	τ (4) [MPa]
3074	450	535	400	72

Because values of stresses calculated on formulas (1)–(4) differ significantly from each other significantly, the verification FEM analysis was conducted.

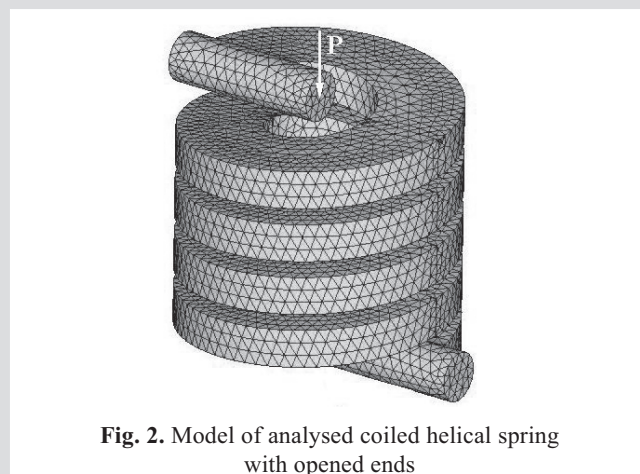


Fig. 2. Model of analysed coiled helical spring with opened ends

In Figure 2 the mesh of spring with opened end-coils is shown. The following conditions were simulated: force P acts exactly along main axis of the spring, arrow-head points the node where force is located. This node has only one degree of freedom – translation along main axis. Equivalent node on the opposite side of spring has zero degrees of freedom – it works as an articulated joint. In Figure 3 the distribution of reduced stresses according to Huber-Mises-Hencky (HMH) hypothesis on the lump of the model is presented.

Twisting rod of rectangular section reveals torsional stresses which achieve the biggest value in the middle of longer side of rod's section [6]. Figure 3 shows that the highest reduced stresses appear in the middle of inner surface of wire (which is the shorter side of rod's section) and their value equals 818 MPa. Considering the influence of bending, compressing, shearing and twisting on stress distribution in spring and transforming formula (5) for reduced stresses according to Huber-Mises-Hencky hypothesis, the maximum value of torsional stresses in analysed model was calculated.

$$\sigma_{HMH} = \sqrt{(\sigma_g - \sigma_c)^2 + 3(\tau_{SH} + \tau_{FEM})^2} \quad (5)$$

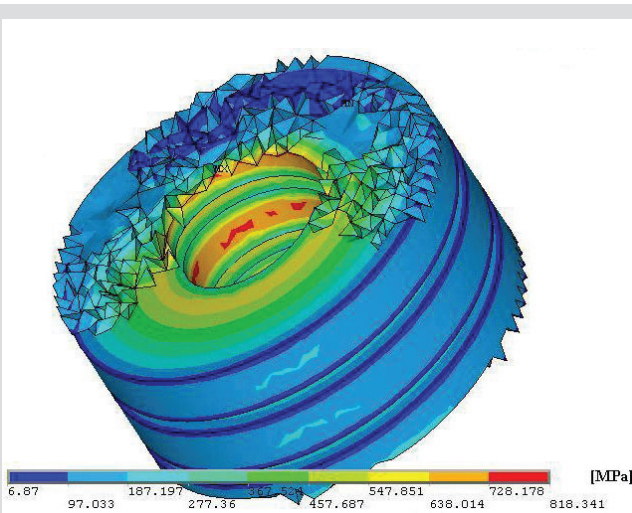


Fig. 3. The distribution of reduced stresses according to HMM hypothesis in a spring loaded with axial force P

- Bending stresses according to strongly curved rods theory in application to rods of rectangular section [6]:

$$\sigma_g = \frac{0.5PD \sin \gamma}{F \left(R - \frac{b}{\ln \left(\frac{\rho_2}{\rho_1} \right)} \right)} \left(\frac{r - \rho_1}{\rho_1} \right), \quad \gamma = a \tan \frac{P}{\pi D},$$

where:

- $F = axb$ – wire section,
- R – gravity centre of wire cross - section,
- ρ_1 and ρ_2 – inner and outer radius of spring respectively,
- r – radius of indifferent layer of wire.

- Compressing stresses (having the same direction as bending stresses)

$$\sigma_c = \frac{P \sin \gamma}{ab}$$

- Shear stresses

$$\tau_{SH} = \frac{P \cos \gamma}{ab}.$$

- Torsional stresses τ_{FEM} value is unknown.

Substituting the values from formulas given above to (5) and knowing that σ_{HMH} equals 818 MPa it was calculated that torsional stresses in the place of maximum stress concentration according to FEM analysis equal

$$\tau_{FEM} = 471 \text{ MPa}.$$

Calculations presented above reveal that formula (1) given by W. Korewa and formula (2) given by S. Żukowski give values comparable with FEM analysis, formula (3) gives values not significantly lower than the ones from FEM, whilst the formula (4) given by E.I. Rivin gives values a few times lower than the FEM ones.

Figure 4 shows reduced stresses distribution in spring with parameters given above, loaded with the same compressing force but with closed final coils as it is shown in the Figure 1. Additionally the real mounting conditions that make impossible mutual changes of angular positions of spring heads were simulated.

Comparing Figure 3 and Figure 4 one can confirm that the way of mounting ends of spring has a big influence on stress distribution in spring. In analysed case maximum stresses in spring machined from cylindrical sleeve, with closed final coils are about 20% higher than in spring coiled from wire, with opened final coils. The highest stress concentration appears in the neighbourhood of round surface of groove, where the coils begin.

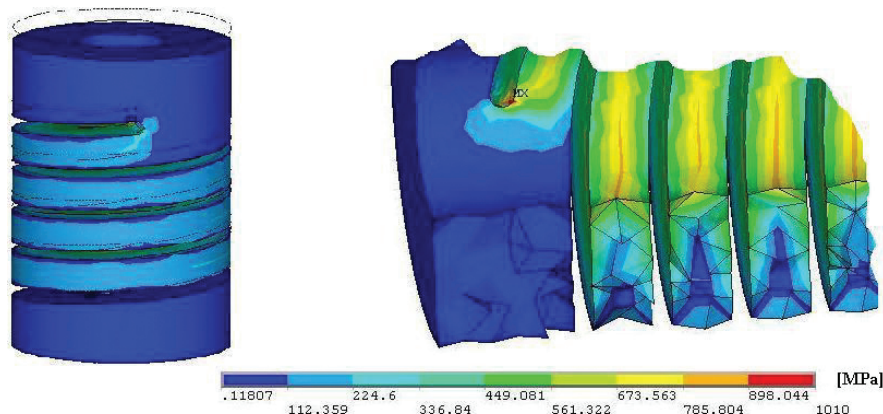


Fig. 4. Analysed spring model with closed final coils and the fragment of elemental section with marked place of highest HMM stress concentration (point MX)

3. CONCLUSIONS

Comparing stress values calculated according to the references with values from FEM analysis one can confirm that the existing in literature methods of stress calculation in helical springs have diverse accuracy. Particularly formula (4) presented in [3] gives results incomparable both with a theory and FEM experiment. The springs similar to the ones presented in article are used especially when high stiffness is needed. To achieve high stiffness of the helical spring the wire's relative radius of curvature has to be very small. The smaller relative radius of curvature the bigger inequality of stress distribution in section and higher maximum stresses. This factor is not taken into consideration in formula (4). Formula (1) gives results to a low degree smaller than values calculated according to FEM analysis. On the contrary formula (2) presented by S. Żukowski gives results somewhat higher than results from FEM experiment, however it is profitable situation because it increases safety factor. The results presented above indicate that formulas (1) and (2) have high accuracy in case of helical springs with opened ends coiled from wire, however using it in case of springs with closed ends machined from cylindrical sleeves does not give very precise results. As it is shown in the Figure 4 maximum reduced stresses appear on the edge of wire (point MX). According to the Saint-Venant theory torsional

stresses decrease to zero on the edges of twisted rod with rectangular cross – section. It means that maximum axial force loading the spring similar to the ones shown in the Figure 1 can not be calculated according to torsional stresses. Another criterion defining maximum load has to be found. Because of complex state of stress appearing in point MX this criterion should be based on strength theory. Therefore it allows to confirm that further structural analysis of helical springs with closed ends needs to be conducted in order to find formulas allowing to calculate maximum reduced stress with accuracy high enough for practical applications.

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