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A Bi-Objective Mixed Integer Program for Supplier Selection**

1. Introduction

In make-to-order environment customer-oriented manufacturers should be prepared to produce varieties of products to meet the different customer needs. Each product is typically composed of many common and non-common (custom) parts that can be sourced from different approved suppliers with different supply capacity. An important issue is how to best allocate the orders for parts among various part suppliers to fulfill all customer orders for products and to achieve a high customer service level at a low cost. The decision maker needs to decide from which supplier to purchase parts required to complete each customer order. The decision is based on price, quality (defect rate) and reliability (on time delivery) criteria that may conflict each other, e.g. the supplier offering the lowest price may not have the best quality or the supplier with the best quality may not deliver on time.

In spite of the importance of supplier selection and order allocation problems, the decision making is not sufficiently addressed in the literature (for a recent review, see Aissaoui *et al.* (2007), in particular for make-to-order manufacturing environment, e.g. Murthy *et al.* (2004), Sawik (2005). Basically, the authors distinguish between supplier selection with single or multiple sourcing, where each supplier can fully meet all requirements or none of the suppliers is able to satisfy the total requirements, respectively. The vast majority of the decision models are mathematical programming models either single objective, e.g. Kasilingam and Lee (1996) or multiple objectives, e.g. Weber and Current (1993), Demirtas and Ustun (2008), Ustun and Demirtas (2008).

The models developed for supplier selection and order allocation can be either single-period models (e.g. Demirtas and Ustun (2008)) that do not consider inventory management

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or multi-period models (e.g. Ustun and Demirtas (2008)), which consider the inventory management by lot-sizing and scheduling of orders. Since common parts can be efficiently managed by material requirement planning methods, this research is focused on custom parts that can be critical in make-to-order manufacturing. For custom-engineered products no inventory of custom parts can be kept on hand. Instead, the custom parts need to be requisitioned with each customer order and hence the custom parts inventory need not to be considered.

This paper presents bi-objective mixed integer programming model for supplier selection in make-to-order manufacturing for a static portfolio of suppliers, that is for the allocation of orders for parts among the suppliers with no timing decisions. In contrast to the dynamic portfolio, which is is the allocation of orders among the suppliers combined with the allocation of orders among the planning periods. In this paper, however, the risk of unreliable supplies for the selected static portfolio is measured by the maximum number of periods in which the average defect rate or late delivery rate is unacceptable.

The paper is organized as follows. In Section 2 description of the supplier selection problem in make-to-order manufacturing is provided. A bi-objective mixed integer program for the supplier selection problem is presented in Section 3. Numerical examples and some computational results are provided in Section 4, and final conclusions are made in the last section.

2. Problem description

In the supply chain under consideration various types of products are assembled by a single producer to satisfy customer orders, using custom parts purchased from multiple suppliers.

Each supplier can provide the producer with custom parts for all customer orders. However, the suppliers have different limited capacity and, in addition, differ in price and quality of offered parts and in reliability of on time delivery of parts. Let $I = \{1, ..., m\}$ be the set of m suppliers and $J = \{1, ..., n\}$ the set of n customer orders for the products, known ahead of time. Each order $j \in J$ is described by the quantity s_j of required custom parts. The planning horizon consists of h delivery periods (e.g. days or weeks) and let $T = \{1, ..., h\}$ be the set of delivery periods. Let c_i be the capacity of supplier $i \in I$, o_i – cost of ordering parts from supplier i, p_{ij} – purchasing price of part for customer order j from supplier i, q_{it} – the expected defect rate of supplier i in period t and r_{it} – the expected late delivery rate of supplier i in period t. The rates q_{it} and r_{it} are based on past observations.

The decision maker needs to decide from which supplier to purchase custom parts required for each customer order to achieve a low unit cost at a high quality and reliability of supplies.

Table 1
Notation

		Indices
i	=	supplier, $i \in I = \{1, \ldots, m\}$
j	=	customer order, $j \in J = \{1, \ldots, n\}$
t	=	delivery period, $t \in T = \{1, \ldots, h\}$
		Input Parameters
c_i	=	capacity of supplier i
o_i	=	cost of ordering parts from supplier i
p_{ij}	=	price of part for customer order j purchased from supplier i
q_{it}	=	expected defect rate of supplier i in period t
r_{it}	=	expected late delivery rate of supplier i in period t
s_j	=	number of parts to be purchased for customer order j
D	=	$\sum_{j \in J} s_j$ – total demand for parts
$\overline{q}$	=	the largest acceptable average defect rate of supplies
$\overline{r}$	=	the largest acceptable average late delivery rate of supplies
$\overline{v}$	=	the maximum allowed number of periods with the average defect rate or average late delivery rate of supplies greater than $\overline{q}$ or $\overline{r}$, respectively

3. Problem formulation

In this section a bi-objective mixed integer program is proposed for single-period supplier selection and order allocation problem, i.e. for determining a static portfolio of suppliers.

The static portfolio of suppliers is defined as

$$(x_1, \ldots, x_m),$$

where

$$\sum_{i \in J} x_i = 1$$

and $0 \le x_i \le 1$ is the fraction of the total demand for parts ordered from supplier i.

When deciding on static portfolio of suppliers it is assumed that the orders for all parts are simultaneously placed on selected suppliers (e.g. at time 0), and the parts from different suppliers are simultaneously delivered. Therefore, in this case the allocation of orders for parts among the suppliers is not combined with the allocation of orders among the planning periods. Nevertheless, given past observation, the static portfolio should be checked over time horizon against the risk of too low quality of purchased parts (too high defect rate) and too low reliability of supplies (too high late delivery rate). We assume that the risk can be measured by the number of periods in which the average defect rate or the late delivery rate of supplies are unacceptable.

Table 2
Problem variables

v_t	=	1 if in period t the average defect rate or the average late delivery rate of supplies for the selected portfolio is, respectively greater than $\overline{q}$ or greater than $\overline{r}$, otherwise $v_t = 0$ (portfolio selection variable)
x_i	=	the fraction of total demand for parts ordered from supplier i (order allocation variable)
y_i	=	1 if an order for parts is placed on supplier i; otherwise $y_i = 0$ (supplier selection variable)
z_{ij}	=	1 if parts required for customer order j are ordered from supplier i; otherwise $z_{ij} = 0$ (customer order assignment variable)

Denote by $\overline{q}, \overline{r}$ and $\overline{v}$ the maximal acceptable defect rate of portfolio, the maximal acceptable late delivery rate of portfolio and the maximum number of periods in which the average defect rate or the average late delivery rate of the portfolio can be above the threshold $\overline{q}$ or $\overline{r}$, respectively.

We assume that the decision maker is willing to accept only portfolios for which the number of periods with the average defect rate greater then $\overline{q}$ or with the average late delivery rate greater than $\overline{r}$ is not greater than the threshold $\overline{v}$.

The quality of the static portfolio can be measured by the following two criteria

$$f_1 = \sum_{i \in I} o_i y_i / D + \sum_{i \in I} \sum_{j \in J} o_{ij} s_j z_{ij} / D \tag{1}$$

$$f_2 = \sum_{i \in I} \sum_{t \in T} (q_{it} + r_{it}) x_i / h \tag{2}$$

where f_1 is the average ordering and purchasing cost per part, and f_2 is the average defect and late delivery rate.

The bi-objective mixed integer program **SP** for the supplier selection problem is formulated below.

Model SP: *Static Portfolio: single-period supplier selection and order assignment*
Minimize

$$f = [f_1, f_2] \tag{3}$$

subject to

1. *Order assignment constraints:*
   – for each customer order required parts are supplied by exactly one supplier,
   – for each supplier the total quantity of ordered parts cannot exceed its capacity,

$$\sum_{i \in I} z_{ij} = 1; \quad j \in J \tag{4}$$

$$\sum_{j \in J} s_j z_{ij} \le c_i; \quad i \in I \tag{5}$$

2. *Portfolio selection constraints:*
   – the portfolio definition constraint (note that the order allocation variable x_i is an auxiliary variable determined by z_{ij}),
   – for each period t, the portfolio with average defect rate greater than $\overline{q}$ is not acceptable,
   – for each period t, the portfolio with average late delivery rate greater than $\overline{r}$ is not acceptable,
   – the portfolio can be unacceptable in at most $\overline{v}$ periods,
   – the order for parts is placed on supplier i if for at least one customer order the required parts are ordered from supplier i,

$$x_i = \sum_{j \in J} s_j z_{ij} / D; \quad i \in I \tag{6}$$

$$v_t \ge \frac{\sum_{i \in I} q_{it} x_i - \overline{q}}{1 - \overline{q}}; \quad t \in T \tag{7}$$

$$v_t \ge \frac{\sum_{i \in I} r_{it} x_i - \overline{r}}{1 - \overline{r}}; \quad t \in T \tag{8}$$

$$\sum_{t \in T} v_t \le \overline{v} \tag{9}$$

$$y_i \ge x_i; \quad i \in I \tag{10}$$

$$y_i \le \sum_{j \in J} z_{ij}; \quad i \in I \tag{11}$$

3. *Non-negativity and integrality conditions*

$$v_t \in \{0,1\}; \quad t \in T \tag{12}$$

$$x_i \ge 0; \quad i \in I \tag{13}$$

$$y_i \in \{0,1\}; \quad i \in I \tag{14}$$

$$z_{ij} \in \{0,1\}; \quad i \in I,\ j \in J \tag{15}$$

Constraints (7) and (8) prevent the choice of portfolios whose average defect rate or whose average late delivery rate is above the fixed threshold $\overline{q}$ (7) or $\underline{r}$ (8), respectively. For each period t such that the average defect rate $\sum_{i \in I} q_{it} x_i$ is greater than the largest acceptable rate $\overline{q}$ (7) or the average late delivery rate $\sum_{i \in I} r_{it} x_i$ is greater than the largest acceptable rate $\overline{r}$ (8), $v_t = 1$. Then all periods with the average defect rate or with the average late delivery rate above the threshold are summed up in (9). If the result is greater than $\overline{v}$, then the portfolio is infeasible.

3.1. Reference point based scalarizing program

Let $\underline{f} = \left(\underline{f}_1, \underline{f}_2\right)$ be a reference point in the criteria space such that $\underline{f} < f_l, l = 1, 2$ for all feasible solutions satisfying (1),(2),(4)–(15), and denote by α a small positive value. The non-dominated solution set of the bi-objective program **SP** can be found by the parameterization on λ the following mixed integer program **SP$\boldsymbol{\lambda}$**.

Model SP$\boldsymbol{\lambda}$

$$\min\{\delta + \alpha(f_1 + f_2)\} \tag{16}$$

subject to (1),(2),(4)–(15) and

$$\lambda\left(f_1 - \underline{f}_1\right) \le \delta \tag{17}$$

$$(1-\lambda)\left(f_2 - \underline{f}_2\right) \le \delta \tag{18}$$

$$\delta \ge 0 \tag{19}$$

where $0 \le \lambda \le 1$.

Program **SPλ** is based on the augmented λ – weighted Tchebycheff metric $\min\left\{\lambda\left|f_1-\underline{f_1}\right|,(1-\lambda)\left|f_2-\underline{f_2}\right|\right\}$, e.g. Steuer (1986).

4. Computational examples

In this section some computational examples are presented to illustrate possible applications of the proposed mixed integer programming approach. The following parameters have been used for the example problems:

– h, the planning horizon, was equal to 30;
– m, the number of suppliers, was equal to 20;
– n, the number of customer orders, was equal to 100;
– o_i, the cost of ordering parts from supplier i, was equal to 5000 for each supplier i;
– p_{ij}, the unit price of parts required for each customer order j, purchased from each supplier i was uniformly distributed over [10, 15] and reduced by factor $(1-\max_{t\in T}(q_{it}+r_{it}))$ to get a lower price for parts from supplier with higher defect and late delivery rates;
– q_{it} and r_{it}, the expected defect rate and the expected late delivery rate of each supplier i in every period t, was uniformly distributed over (0, 0.08) and (0, 0.10), respectively;
– $\overline{q}$ and $\overline{r}$, the largest acceptable average defect rate and late delivery rate, was equal to 0.05 and 0.06, respectively;
– $\overline{v}$, the maximum allowed number of periods with the average defect or late delivery rates greater than, respectively $\overline{q}$ = 0.05 or $\overline{r}$ = 0.06, was equal to 0, 1 or 2;
– s_j, the numbers of required parts for each customer order, were integers uniformly distributed over [100, 5000];
– c_i, the capacity of each supplier i, was equal to $\left\lceil 2\sum_{j\in J} s_j/m\right\rceil$.

The computational results and a subset of non-dominated solutions for selected values of parameter λ are presented in Table 3. The size of the mixed integer program **SPλ** is represented by the total number of variables, Var., number of binary variables, Bin., and number of constraints, Cons. The last two columns of the tables present the solution values and CPU time in seconds required to prove optimality of the solution. The computational experiments were performed using AMPL programming language and the CPLEX v.11 solver (with the default settings) on a laptop with Pentium IV processor running at 1.8 GHz and with 1 GB RAM. The solver was capable of finding proven optimal solutions for all examples in a very short CPU time.

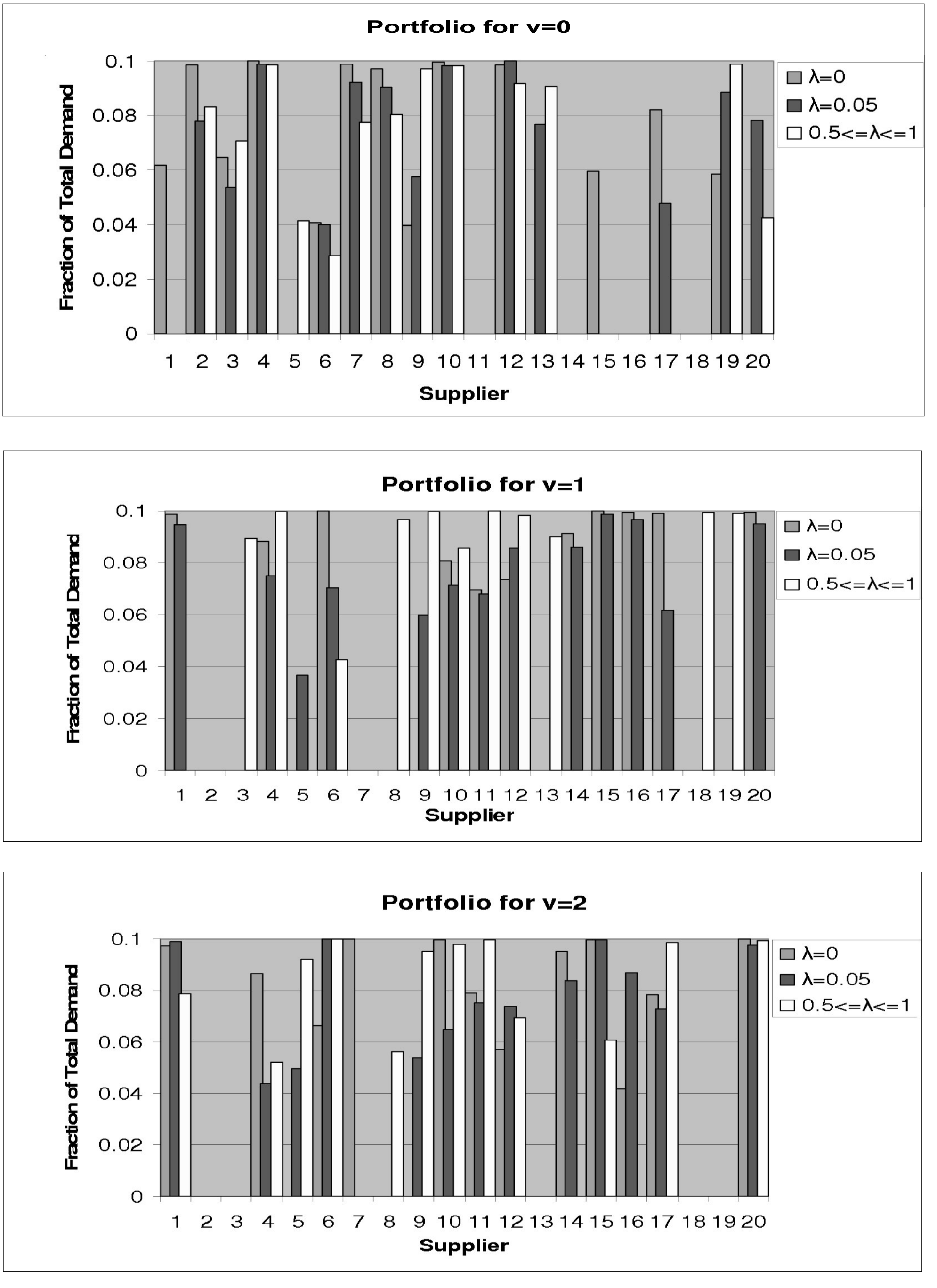

Fig. 1. Optimal portfolios for non-dominated solutions

Table 3
Computational results

$\bar{v}$	Var.	Bin.	Cons.	f_1, f_2 [(a)], No. of Suppliers [(b)]	CPU [(c)]
				$\lambda = 0$	
0	2041	2020	242	8.401, 0.089, 13	459
1	2071	2050	243	8.386, 0.086, 11	262
2	2071	2050	243	8.424, 0.085, 12	48
				$\lambda = 0.05$	
0	2041	2020	243	8.390, 0.090, 13	746
1	2071	2050	244	8.328, 0.087, 13	338
2	2071	2050	244	8.321, 0.087, 13	33
				$\lambda = 0.5$	
0	2041	2020	243	8.363, 0.091, 13	239
1	2071	2050	244	8.304, 0.092, 11	72
2	2071	2050	244	8.284, 0.089, 12	73
				$\lambda = 0.95$	
0	2041	2020	244	8.363, 0.091, 13	134
1	2071	2050	244	8.304, 0.092, 11	460
2	2071	2050	244	8.284, 0.089, 12	22
				$\lambda = 1$	
0	2041	2020	242	8.363, 0.091, 13	232
1	2071	2050	243	8.304, 0.092, 11	53
2	2071	2050	243	8.284, 0.089, 12	102

(a) f_1, f_2 – the average ordering and purchasing cost per part, the average defect and late delivery rate, respectively ($\underline{f_1} = 8$, $\underline{f_2} = 0{,}07$).

(b) the number of selected suppliers.

(c) CPU seconds for proving optimality on a PC Pentium IV, 1.8 GHz, RAM 1 GB/CPLEX 11.

Notice that only three non-dominated solutions (f_1, f_2) were found for each $\bar{v}$: (8.401, 0.089), (8.390, 0.090), (8.363, 0.091) for $\bar{v} = 0$, (8.386, 0.086), (8.328, 0.087), (8.304, 0.092) for $\bar{v} = 1$, and (8.424, 0.085), (8.321, 0.087), (8.284, 0.089) for $\bar{v} = 2$. In particular, only one solution for each $\bar{v}$ was found for $0.5 \le \lambda \le 1$.

Figure 1 shows the optimal allocation of demand for parts among the suppliers for the three non-dominated solutions and the three risk levels of unreliable supplies, represented by the maximum number of periods $\bar{v} = 1, 2, 3$ in which the average defect rate or late delivery rate of supplies can be unacceptable. The number of selected suppliers varies between 11 and 13, and the allocated fraction of the total demand for parts varies between $x_i = 0.028$ for the lowest risk level $(\overline{v} = 0)$ and $x_i = 0.1$ for the highest risk level $(\overline{v} = 2)$.

5. Conclusion

The problem of optimal allocation of orders for parts among a set of approved suppliers in make-to-order manufacturing has been modeled as a bi-objective mixed integer program. In the model, the risk level of unreliable supplies that the decision maker is disposed to accept is measured by the maximum number of periods, in which the average defect rate or late delivery rate can be unacceptable.

The limited computational experiments have indicated that the proposed approach requires a short CPU time to find the optimal solution in a static case, where all customer orders are known ahead of time. The last assumption can be relaxed, and the approach can also be used in a dynamic case where orders arrive irregularly over time.

In make to order environment, in which custom parts are typically ordered in small lot sizes, supplier may sometimes offer discount that depends on total value of sales volume (business volume) or on total quantity of ordered parts. The proposed model can also be enhanced for discount environment.

References

[1] Aissaoui N., Haouari M., Hassini E., *Supplier selection and order lot sizing modeling: A review.* Computers & Operations Research, 34, 2007, 3516–3540.
[2] Demirtas E.A., Ustun O., *An integrated multiobjective decision making process for supplier selection and order allocation.* Omega. The International Journal of Management Science, 36, 2008, 76–90.
[3] Kasilingam R.G., Lee C.P., *Selection of vendors a mixed-integer programming approach.* Computers and Industrial Engineering, 31, 1996, 347–350.
[4] Murthy N.N., Soni S., Ghosh S., *A framework for facilitating sourcing and allocation decisions for make-to-order items.* Decision Sciences, 35, 2004, 237–259.
[5] Sawik T., *A cyclic versus flexible approach to materials ordering in make-to-order as-embly.* Mathematical and Computer Modelling, 42, 2005, 279–290.
[6] Steuer R.E., *Multiple Criteria Optimization: Theory.* Computation and Application, 1986 (Wiley: New York).
[7] Ustun O., Demirtas E.A., *An integrated multi-objective decision making process for multi-period lot sizing with supplier selection.* Omega. The International Journal of Management Science, 36, 2008, 509–521.
[8] Weber C.A., Current J.R., *A multiobjective approach to vendor selection.* European Journal of Operational Research, 68, 1993, 173–184.