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GENERALISED STRUCTURE AND EQUIVALENT CIRCUITS OF ELECTRICAL MACHINES**

SUMMARY

The paper deals with the problem of structure of electrical machine as electromechanical converter of energy. The samples of existing representation of electrical machine structure and critical point of view are presented. The generalised structure on a basis of generalised energy converters is proposed. According to this structure, the principles of formation of entire equivalent circuits (with direct reflection of elements booth of electrical and mechanical systems) are studied. Presented generalised approach can help to analyse various regimes of electrical machines.

Keywords: *electromechanical energy conversion, theory of electrical machines, circuits theory*

UOGÓLNIONA STRUKTURA I OBWODY ZASTĘPCZE MASZYN ELEKTRYCZNYCH

Artykuł poświęcony jest problemowi struktury maszyny elektrycznej jako elektromechanicznego przetwornika energii. Podane zostały przykłady obecnej reprezentacji maszyn elektrycznych wraz z ich krytyczną oceną. Zaproponowano uogólnioną strukturę na podstawie uogólnionego przetwornika energii. Przestudiowano podstawy formułowania wszystkich obwodów zastępczych zgodnie z tą strukturą (wraz z bezpośrednimi relacjami pomiędzy elementami tworzącymi podsystemy elektryczny i mechaniczny). Zaprezentowane ogólne podejście jest pomocne w analizie wielu ograniczeń w maszynach elektrycznych.

Słowa kluczowe: *elektromechaniczne przetwarzanie energii, teoria maszyn elektrycznych, teoria obwodów*

1. INTRODUCTION

Each electrical machine is an electrmechanical energy converter. Therefore always we can distinguish *electrical* system (in fact, the *electromagnetic* system, which mostly is named as *electrical* one) and *mechanical* system in structure of electrical machine. Both systems are analogous each other by their elements of energy storage of two kinds (kinetic and potential energies), and energy dissipating elements.

However, there is great variety of treatment of the structure of electrical machines and presentation of their equivalent circuits, as well as of the process of energy conversion. Therefore this paper is dealt with these prolems.

The mechanical system is of two main kinds: rotational and translational (linear). The analogies of values of these systems are well known, therefore only relationships of translational system are presented in this paper.

2. GENERALISED STRUCTURE OF ELECTRICAL MACHINES

2.1. View of traditional structures

Fairly traditional structure of electrical machine is presented in Figure 1 [1]. Because this structure is adapted to analyse energy balance, the linkages between structure components express flows of energy. In the coupling block we have only storage of energy, without dissipation.

We can see similar presentation of the electrical machine structure in Figure 2 [2]. However, the treatment of energy losses presents essential difference to compare with the former structure. As we can see, here is pointed out the "field loss"; the "electrical loss" somehow is indicated in the linkage, while "mechanical loss" is treated as intrinsic property of the mechanical system.

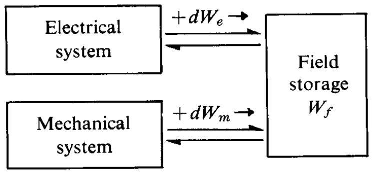

Fig. 1. Diagram of energy balance [1]

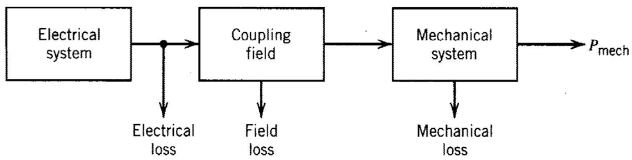

Fig. 2. Structure of electrical machine [2]

In the structure of Figure 3 [3], the electrical system (block *A*) and mechanical system (block *C*) are considered with corresponding losses, but the coupling block *B* is without losses. In fact, the blocks are presented as two-port elements of circuit, designating corresponding power components in the "conductors" and between the "clamps".

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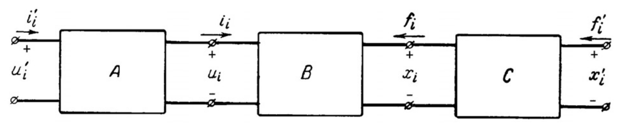

Fig. 3. Structure of electrical machine [3]

Treatment of the structure of electromechanical energy converter is very similar in Figure 4 [4], though the dissipative electrical and mechanical elements are somehow inserted in the linkages of bolcks. The middle-coupling block also is considered with losses (as in the case of Fig. 2).

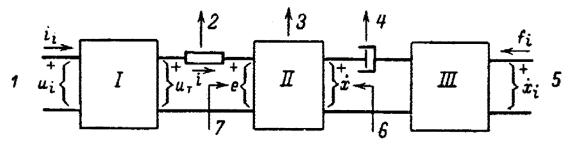

Fig. 4. Structure of electrical machine [4]

There is other treatment of two-port elements as electromechanical transducers, when such element expresses only relationships of variables, but the storage of energy in such electromechanical converter is not reflected (for example, Fig. 5 [5]).

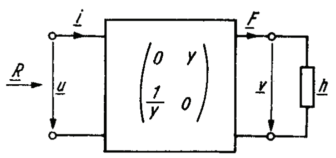

Fig. 5. Two-port element as electromechanical transducer [5]

In fact the simplified structure of electrical machine is also presented as two-port in Figure 6 [6]. The representation is unnatural by its variables. If the mechanical variables torque M and velocity of rotation n are components of mechanical power, the frequency of voltage f is any power component (coupled with the voltage U). This illogical choice of variables is probably caused by imperfection of terms, because in Russian sometimes the *velocity of rotation* is named as *frequency of rotation.*

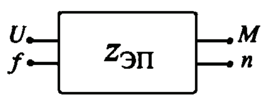

Fig. 6. Simplified structure of electrical machine [6]

There is quit other presentation of the structure of electromechanical energy converter. For example, only relation between electrical and magnetical parts is accentuated in the structure (Fig. 7, [7]), while the mechanical system remains off the map.

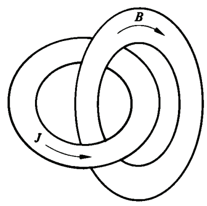

Fig. 7. Schematic relation of electrical and magnetical parts [7]

Thus, the proposed structures of electrical machines are not only different, but sometimes they are contradicting one another.

2.2. Elements of energy conversion

In spite of variety of proposed representation of structures of electrical machines, in all cases one tries to reflect element in which the conversions of energy occurs. The energy W of any system can be expressed through corresponding quantities of energy. These generalised quantities are following: system co-ordinate q, rate of variation of the co-ordinate (named as extensive component *ext*), intensive component of the energy *int*. It should be note that quantities, here presented as power components *int* and *ext*, which product yield the power, are still called by different names: *effort* and *flow*; *potential* and *intensity*; *across* and *through*, though origin and sense these terms are different.

We are postulated following generalised definition of energy conversion [8]:

energy conversion is feasible, if we have an element of energy storage (an accumulator) at least with two different terminals (that is, with the terminals transferring energies of different nature).

The generalised scheme of such element of energy conversion (further named *converter*) is depicted in Figure 8a.

The electromechanical energy conversion is accomplished with storage of energy either in the magnetic or electric field. Consequently, the *magnetomechanical* and *electromechanical* converters could be distinguished, as it is shown in Figures 8b and 8c.

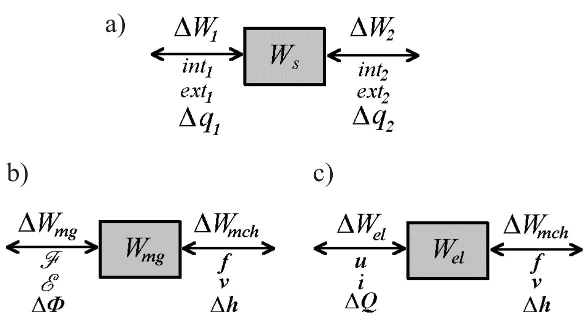

Fig. 8. Generalised scheme of energy converter (a), magnetomechanical (b) and electromechanical (c) converters F – magnetmotive force, E – rate of variation of the magnetic flux Φ, u – voltage, I – current, Q – electric charge, f – mechanical force, ν – velocity, h – mechanical co-ordinate

2.3. Proposed generalised structure

As it was marked out in the introduction, the electromagnetic and mechanical systems are analogous from the standpoint of two kinds of energy storage in their separated parts (subsystems). First of all, the main systems (electromagnetic and mechanical) must be reflected in the structure of electrical machine. Corresponding converters must connect these systems, as it is shown in Figure 9.

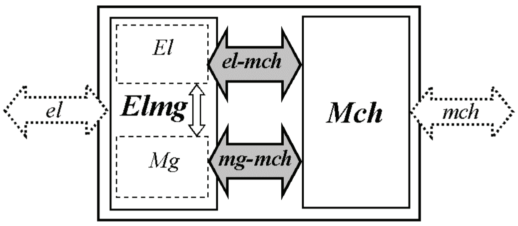

Fig. 9. Generalised structure of electrical machine

The electromagnetic system (*Elmg* in Fig. 9) can be divided into two subsystems (electrical *El* and magnetical *Mg*). Between these subsystems the corresponding connection exists, which is marked out by the vertical arrow. In principle, the mechanical system *Mch* also could be divided in the analogous subsystems (according to elements with kinetic and potential energy storage), but such separation is not topical analysing electrical machine.

If an electrical machine is based on the *magnetomechanical* converters, we have the *inductive* machine. Of course, all conventional and majority of special electrical machines are *inductive* ones. Concretised variants of structure of the *inductive* electrical machine are depicted in Figure 10.

a) b)

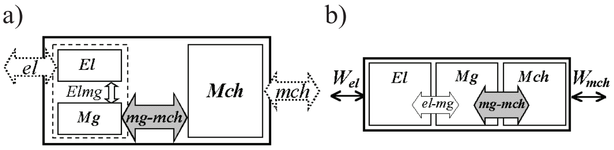

Fig. 10. Structure of *inductive* electrical machine

In analogous way the structure of *capacitive* (that is, machine with storage of energy in electrical field) also could be concretised. Possibility to analyse both *inductive* and *capacitive* electrical machines in unified mode is the main advantage of the proposed presentation of generalised structure of electrical machine.

3. EQUIVALENT CIRCUITS OF ELECTRICAL MACHINES

3.1. Traditional equivalent circuits and principle of entire equivalent circuits

The method of equivalent circuits analysing electrical machines is well known. As a rule, the equivalent circuits contain directly the parameters of machine's electromagnetic system, that is, the parameters of machine's electric circuit. However, the parameters of mechanical circuit, as a rule, are not reflected directly in the traditional equivalent circuits.

Meanwhile, in any case, we have internal mechanical circuit of electrical machine (e.g. consisting of inertia and dissipative element of mechanical losses), and external mechanical circuit, that is, the load of electrical motor in the drive. Therefore naturally a question arises, is it possible to form an integral equivalent circuit (*entire equivalent circuit*) with parameters of machine's electric and mechanical circuits? If these both circuits of the real electrical machine are linear, remains the integral equivalent circuit linear too?

It is to note that sometimes such entire equivalent circuits are analysed (e.g. [7, 9, 10]), but an exhaustive answer to mentioned above questions is not defined jet.

3.2. Instantaneous parameters of circuit elements

Principles and variants of entire equivalent circuits of electrical machines could be studied on a basis of the generalised structure of electrical machines (Fig. 9) using conception of the instantaneous parameters of circuit elements.

The instantaneous parameters of circuit elements are expressed as ratio of instantaneous power components of elements. For instance, in general case the instantaneous electrical resistance $z(t)$ and conductance $y(t)$ are presented by corresponding ratio of electrical power components

$$z(t) = u(t)/i(t),\ y(t) = i(t)/u(t) \qquad (1)$$

In analogous way could be expressed corresponding instantaneous mechanical parameters

$$Z(t) = f(t)/v(t),\ Y(t) = v(t)/f(t) \qquad (2)$$

Typical expressions of dissipative and conservative elements of electrical and mechanical systems are presented in Table 1, vhere: L – inductivity, C – capacity, m – mass, k – rigidity.

Table 1

Instantaneous circuits elements

System	Dissipatives	Conservatives			
		Inertial		Potential	
Electrical	$z_r(t) = r$	$z_L(t)$	$L \cdot \frac{\frac{di(t)}{dt}}{i(t)}$	$z_C(t)$	$\frac{1}{C} \cdot \frac{u(t)}{\frac{du(t)}{dt}}$
			$L \cdot \frac{u(t)}{\int u(t)\,dt}$		$\frac{1}{C} \cdot \frac{\int i(t)\,dt}{i(t)}$
Mechanical	$Z_R(t) = R$	$Z_m(t)$	$m \cdot \frac{\frac{dv(t)}{dt}}{v(t)}$	$Z_k(t)$	$k \cdot \frac{f(t)}{\frac{df(t)}{dt}}$
			$m \cdot \frac{f(t)}{\int f(t)\,dt}$		$k \cdot \frac{\int v(t)\,dt}{v(t)}$

Principal structure of entire equivalent electric circuit of electrical machine is shown in Figure 11a, where the block *El* presents real electric circuit of the machine, and the block *Mch* presents imported equivalent of mechanical system.

The latter can be concretised as imported electical instantaneous resistance $z_{mch}(t)$ reflecting the parameters of mechanical system (Fig. 11b).

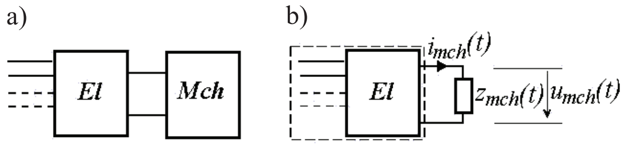

Fig. 11. Principle of entire equivalent electrical circuit

On a basis of the law of energy conservation, we can get a set of identical expressions of this imported instantaneous resistance by parameters of mechanical system and various variables

$$z_{mch}(t) = Z(t) \cdot \left[\frac{\nu(t)}{i_{mch}(t)} \right]^2 = Z(t) \cdot \left[\frac{u_{mch}(t)}{f(t)} \right]^2 \quad (3)$$

$$z_{mch}(t) = Y(t) \cdot \left[\frac{f(t)}{i_{mch}(t)} \right]^2 = Y(t) \cdot \left[\frac{u_{mch}(t)}{\nu(t)} \right]^2 \quad (4)$$

As we can see, the imported equivalent resistance can be expressed though mechanical resistance $Z(t)$ (3) as well as though mechanical conductance $Y(t)$ (4). Consequently, the expressions of the angle brackets (3) can be named as parameter of transformer type connection $K_{tr}(t)$, and as parameter of gyrator type connection in the expression (4).

In general case the parameters $K_{tr}(t)$ and $K_g(t)$ are time functions, but in isolate case one or another parameter can be a constant value. It should signify that the imported equivalent electrical elements of mechanical system are proportional to corresponding mechanical elements. In a case of invariability of the transformer type parameter ($K_{tr}(t) = K_{tr}$ = const.), the imported equivalent circuit conserve the same topology of mechanical circuit, only the parameters of mechanical elements must be multiplied by K_{tr}^2. In a case of invariability of $K_g(t) = K_g$ = const., the imported circuit changes its topology into dual circuit multiplying the mechanical elements by K_g^2. Consequently, only in such a cases linearity of electrical and mechanical systems produces linear entire equivalent circuit.

3.3. Sample of entire equivalent circuit

In Figure 12a the electrical circuit of DC machine is shown. Consider the flux Φ = const. The induced electromotive force e and torque T are following

$$e(t) = K_a\ \Phi\ \omega(t),\ T(t) = K_a\ \Phi\ i_a(t) \quad (5)$$

where:

K_a – constant of the machine,
ω – angular velocity,
i_a – armature current.

The mechanical circuit is shown in Figure 12b.

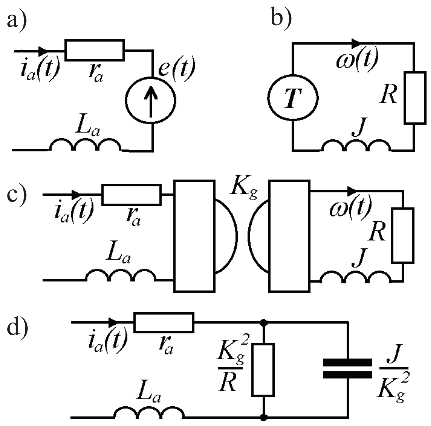

Fig. 12. Electrical circuit of DC machine (a), mechanical circuit (b), equivalent circuit with gyrator (c), entire equivalent circuit (d). J – inertia, R – active mechanical resistance

According to rules presented above, we have expression of the parameter of gyrator type connection

$$z_{mch}(t) = Y(t)\ [K_a\ \Phi] \quad (6)$$

As we can see, the expression in the angle brackets (6) is constant value, therefore

$$[K_a\ \Phi] = K_g(t) = K_g = \text{const} \quad (7)$$

Consequently, the gyrator type connection exists (Fig. 12c), and entire equivalent circuit can be formed (Fig. 12d). In this case, the imported part of this circuit has dual topology of the mechanical circuit (Fig. 12b).

Similar entire equivalent circuit can be formed only for some synchronous AC machines, some capacitive machines, but in majority cases the entire equivalent circuits became non-linear, with complex relationships of parameters of machine systems.

4. CONCLUSIONS

Presented generalised structure of electrical machines and principles formation of the entire equivalent circuits can help to analyse various regimes (steady state as well as transient) of electrical machines.

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He was born in 1936 in Klaipeda (Lithuania). He received his diploma of electrical engineering in 1958 and Dr. Sc. degree in 1963, both from Kaunas Polytechnic Institute (Lithuania), and Dr. Habil. degree from Moscow Energetic Institute in 1980. Since 1963 he started to teach (as Associate Professor and as Professor after 1980) in Klaipeda faculty of Kaunas Polytechnic Institute organising Electrical Department and being a chief of it for many years. In 1991 the department became a division of newly founded Klaipeda University. Since 1996 he also is pursued his research in the framework of S. Kudarauskas personal enterprise. He research activities include electrical machines with peculiarities of mechanical movement (oscillating and other machines) and general problems of energy conversion. He started studies of oscillating electrical machines in 1960. He has published more than 180 technical papers, and 16 patents. In addition, from 1990 (after restitution of the independence of Lithuania) to 1993 S. Kudarauskas has been the diplomatic representative of Lithuanian Republic in Estonian Republic (without interrupting his pedagogical and scientific activities).

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