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Assessment of the Impact of 3D Printed Water-lubricated Cutless Bearings Material on Vibration Parameters

Jędrzej Blaut*

Abstract. The main purpose of the research was to determine the possibilities and experimentally verify the benefits of using thermoplastic materials in a cutless plain bearing. The tested bearings were subjected to equal loads and rotational speeds, using water as the lubricant. Analysis showed that they achieved the best vibration damping at lower speeds, between 600 and 1100 RPM. Comparative studies of bearings made of different materials, such as available on the market bearings made of rubber, bearings printed with a 3D printer from PETG, PLA, ABS or Tribo filament materials, revealed differences in their vibration damping ability and operational stability. Conclusions from the study suggest that higher vibration acceleration may increase the radius of the trajectory, which may affect the function and performance of the bearings. The importance of the operational stability of water-lubricated plain bearings cannot be assessed solely on the basis of the RMS of vibration acceleration or trajectory radius. Both of these parameters are crucial from the user's point of view, especially in the context of various applications such as electric boats.

Keywords: stability of journal bearing, water-lubrificated plain bearings, cutless bearing, bearings with multiple axial grooves, 3D printing, electric boat, cutlass bearing material decision-making

Mathematics Subject Classification: 93E99

JEL Classification: D81, D24, L16

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1. INTRODUCTION

The advancement of electromobility in Europe is pivotal for the future of the marine craft industry, and the use of new thermoplastic materials is introducing innovations

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that have the potential to revolutionize these systems. This trend has the potential to encourage a shift from traditional propulsion solutions towards cleaner and more technologically advanced systems in marine vessels. The current transition plays an important role in reducing the negative impact on marine ecosystems and increasing the energy efficiency of these vessels.

Electric boats have significantly lower noise levels compared to units powered by conventional internal combustion engines. Therefore, it becomes important to use quiet stern bearings to reduce the noise generated by the rotating propeller, which has often been downplayed in boats with internal combustion engines. In addition, these specialized silent bearings enhance reliability by minimizing wear and tear and the risk of mechanical failure in the propulsion structure of electric boats.

Innovative thermoplastic materials and advancements in 3D printing technology offer new possibilities for the development of bearings (Valino et al., 2019). The transition of technology from hobbyist applications to industrial use enables the production of advanced components, including bearings, which can be optimized for performance, durability, and noise reduction. The use of 3D printing in the marine industry allows for rapid prototyping and the production of custom, high-quality parts from advanced materials, which is critical to improving the efficiency, cost savings and energy performance of marine propulsion systems. This synergy between new thermoplastic materials and advanced manufacturing methods such as 3D printing plays an important role in creating more efficient and environmentally friendly solutions for the future of electromobility in the marine sector.

2. WATER-LUBRICATED CUTLESS TYPE PLAIN BEARING APPLICATION IN MARITIME TRANSPORT

Cutless bearings are a type of plain bearings used in watercrafts, especially in the propulsion systems of marine transportation means such as boats, yachts, ships or other vessels.

Cutless bearings are installed in locations with the rotation of the propeller shaft, such as within thrusters, aft of the hull, or in propeller supports. They provide support and reduce friction as the shaft rotates, allowing the vessel to move smoothly. Typical stern bearings are made of bronze or other metal alloys that are resistant to abrasion and corrosion. Modern versions of bearings can also incorporate advanced synthetic materials that are more resistant to abrasion and provide longer machine operation life.

One of the most widely favored materials used in the manufacturing of cutless bearings is nitrile rubber. A study conducted by Cabrera and other authors in 2005 contributed to the understanding of how pressures are distributed in cutless bearings. It was found that in the area of minimum lubricating film thickness, three staves, or three contact points, carried the main part of the load (Cabrera et al., 2005). The research conducted by Wang et al. in 2014 was based on theoretical analyses related to the characteristics of plain bearings. This research work focused mainly on theoretical approaches using mathematical models and numerical simulations to understand the behavior of bearings under different operating conditions, and presented experimental results from a test stand (Wang et al., 2014). The research conducted by Zhou and team focused on the experimental study of water film pressure in water-lubricated rubber bearings with multiple grooves (WLRBMG). As part of the research, a special test rig was designed to measure various parameters of WLRBMGs to further analyze the properties of this type of bearing (Zhou et al., 2017).

Currently, there exists a necessity to integrate theoretical findings with experimental data to enhance the performance characteristics of water-lubricated plain bearings (Blaut & Breńkacz, 2020). The research includes analysis of vibration and the effect of improper shaft alignment on the bearings. In addition, research is being conducted to develop a theoretical model based on experimental data, with the aim of designing cutless bearings efficiently (Smith, 2020). This research work aims not only to better understand the dynamics of bearing operation under water lubrication conditions, but also to develop more precise design methods that take into account the various factors that affect bearing performance and durability.

Research work is also being carried out on polymers for sliding bearings. Research on PEEK material showed an interesting effect of lapping of the polymer material in that the coefficient of friction in the lapped bearing was almost independent of the load (Żochowski et al., 2023). A separate research topic is the study of the wear of polymer bearings lubricated by contaminated water, which is characteristic of vessels (Litwin et al., 2023). There is also a need to analyze the possibility of replacing traditional petroleum-based lubricants by water. This has beneficial effects on the environment, lubricants are an environmental hazard in case of spills and their elimination reduces the need for petroleum. Replacing traditional lubricants with water results in lower friction losses in bearings due to the lower viscosity of water, which reduces energy dissipation in machinery (Wasilczuk et al., 2023).

Figure 1 shows a cutless water bearing, characterized by noise reduction, environmental protection by eliminating oil spills into the seas, and minimized friction, which extends their life. However, their sensitivity to leakage requires regular maintenance and monitoring, which is a limitation. Performance can be affected by environmental conditions, such as water temperature and the presence of contaminants.

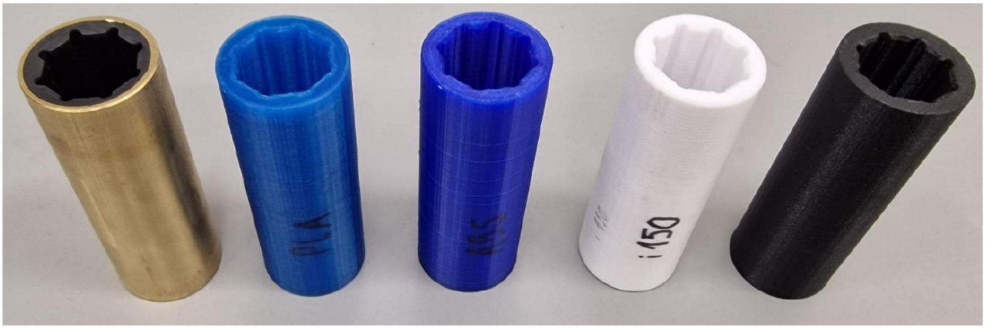

Fig. 1. *Tested water-lubricated plain bearing designs made of rubber liner in bronze pan (commercial solution), PLA, ABS, Tribo filament, PETG*

3. PREPARATION OF CUTLESS TYPE PLAIN BEARINGS FOR TESTING

Experimental studies have focused on the shape and configuration of the flexible liners used in cutless bearings, particularly related to the number of grooves used. Researchers such as Pai and Pai in 2008 conducted experiments to understand how different numbers of grooves affect bearing behavior and performance under water lubrication conditions (Pai & Pai, 2008).

Studies have also been conducted on the effect of scratches on the performance of water-lubricated rubber-lined plain bearings are well known. Experiments showed a significant effect of scratches on the critical load and critical speed of these bearings. According to the results of these experiments (a scratched shaft has a longer motion orbit and a lower equilibrium point compared to a shaft without scratches (Liang et al., 2023).

Fused Deposition Modeling (FDM) 3D printing technology involves the layering of a material, usually a thermoplastic, in a computer-controlled manner, which allows the construction of three-dimensional objects. In the context of the study, four thermoplastic materials were selected to enable 3D printing:

ABS (acrylonitrile butadiene styrene) is a thermoplastic polymer that is distinguished by its mechanical strength and resistance to damage. ABS has good compressive and tensile strength, making it ideal for printing parts with increased strength requirements, such as mechanical parts, utility components or functional prototypes.

PETG (polyethylene terephthalate glycol) is a thermoplastic polymer with high transparency, chemical and mechanical resistance. PETG is relatively easy to print, and its characteristics of strength, flexibility and durability make it popular for packaging, electronic components, and prototypes with higher strength requirements.

PLA (polylactic acid) is a biodegradable polymer of organic origin, most commonly used in 3D printing due to its ease of printing and environmental sustainability. PLA is odorless, emits no harmful substances, and has good strength and plasticity. It is commonly used to print prototype models, decorative elements, and objects that do not require high mechanical resistance.

Tribo filament is an advanced thermoplastic polymer that exhibits unique properties such as high chemical, thermal and mechanical resistance. Tribo filament is used in applications where exceptional properties are required, such as production of industrial parts, machine components, or in applications where high chemical resistance is needed.

Each of these thermoplastic materials has its own unique characteristics and applications, so it was decided to test them for use in cutless bearings. The geometry of the bearings made is based on an outer diameter of 30 mm and an inner diameter of 20.2 mm, and a diameter-to-length ratio of 1:4 which coincides with the general guidelines of flex-lined plain bearings. The bearings, made of thermoplastic materials, were compared with a commercial cutless plain bearing made of bronze pan with rubber lining.

4. TEST STAND DESCRIPTION

The test stand, located in the Machine Diagnostics and Monitoring Systems Laboratory, was used to test sliding bearings. This test fixture was specially adapted to carry out tests at different speeds and loads. Enables the conduction of tests for the diagnostic assessment of plain bearings and the evaluation of their technical condition (Fig. 2).

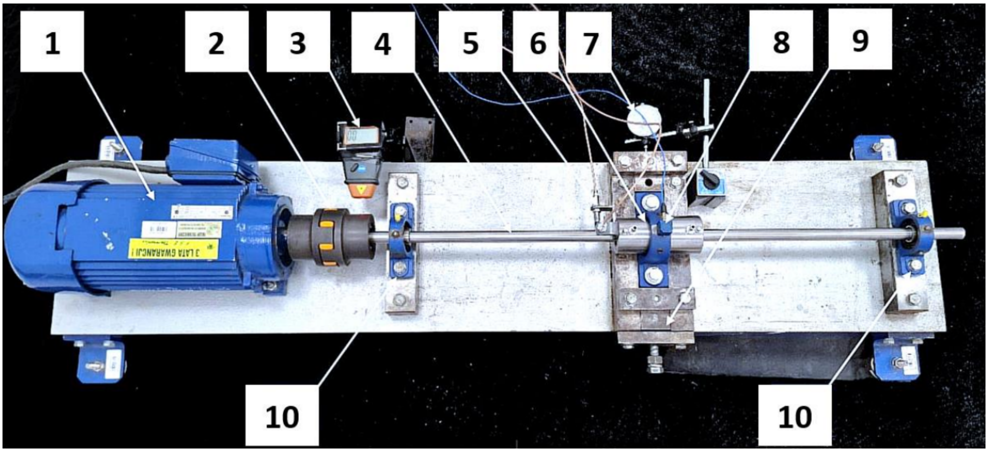

Fig. 2. *Test stand parts: 1 – motor, 2 – clutch, 3 – tachometer, 4 – shaft, 5 – eddy current sensors x, y, 6 – test bearing assembly, 7 – shaft deflection dial sensor, 8 – triaxial accelerometer, 9 – loading system, 10 – rolling bearings*

The bearing is positioned within a housing located at the center of the shaft, between two support bearings. The system is loaded laterally by tightening a screw, moving the test bearing assembly. Based on the deflection distance (center of deflection of the beam), the force acting on the test bearing is determined. The basic elements of this measuring station are:

– a drive system moving the shaft in a rotating motion, consisting of a motor and a clutch,
– two roller bearings supporting the ends of the shaft,
– a tachometer measuring the speed of the shaft,
– a set of test plain bearing with mounted eddy current distance sensors MDS10PO and piezoelectric accelerometer 3-axis PCB-356B08,
– a system loading the test bearing set with a lateral force, equipped with a shaft deflection sensor with a timer.

5. PERFORMANCE TESTS ON PLAIN BEARINGS

Each of the plain bearings tested was subjected to identical tests, under equal conditions of overhung loads and rotational speeds. Lubricating water was supplied through nozzles, completely filling the bearing shell along with the bearing. Lateral loading of the bearing with a force of 40 N, 80 N and 120 N corresponded to shaft deflections of 0.5 mm, 1 mm and 1.5 mm, respectively. The rotational speeds were determined based on typical operating conditions of recreational boats, where speeds do not exceed 3,000 RPM, and the range of typical speeds is between 600 and 1,800 RPM.

Table 1. *RMS values of acceleration and radius of rotational trajectory for 0 N, 40 N, 80 N, and 120 N excitations for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

Rubber	0 N		40 N		80 N		120 N	
RPM	RMS	Radius	RMS	Radius	RMS	Radius	RMS	Radius
600.0	0.02	0.15	0.07	0.06	0.01	0.07	0.08	0.08
1000.0	0.06	0.15	0.08	0.10	0.07	0.10	0.18	0.012
1200.00	0.18	0.28	0.08	0.15	0.25	0.10	0.16	0.11
1500.00	0.08	0.25	0.17	0.15	0.17	0.15	0.09	0.07
1800.00	0.02	0.18	0.09	0.13	1.14	0.18	0.20	0.11
ABS	0 N		40 N		80 N		120 N	
RPM	RMS	Radius	RMS	Radius	RMS	Radius	RMS	Radius
600.00	0.08	0.15	0.13	0.09	0.08	0.06	0.09	0.08
1000.00	0.28	0.15	0.55	0.09	0.20	0.07	0.20	0.07
1200.00	0.29	0.13	0.32	0.13	0.30	0.06	0.17	0.11
1500.00	0.10	0.16	0.31	0.09	0.36	0.11	0.66	0.11
1800.00	0.75	0.13	0.71	0.13	0.60	0.12	0.39	0.14
PETG	0 N		40 N		80 N		120 N	
RPM	RMS	Radius	RMS	Radius	RMS	Radius	RMS	Radius
600.00	0.07	0.09	0.04	0.04	0.12	0.07	0.03	0.04
1000.00	0.07	0.07	0.13	0.07	0.03	0.06	0.18	0.04
1200.00	0.06	0.07	0.07	0.07	0.25	0.10	0.11	0.05
1500.00	0.16	0.05	0.27	0.06	0.23	0.06	0.31	0.04
1800.00	0.09	0.06	0.16	0.05	0.32	0.05	0.34	0.05
PLA	0 N		40 N		80 N		120 N	
RPM	RMS	Radius	RMS	Radius	RMS	Radius	RMS	Radius
600.00	0.05	0.11	0.06	0.06	0.06	0.05	0.05	0.05
1000.00	0.13	0.10	0.19	0.06	0.02	0.06	0.17	0.04
1200.00	0.22	0.07	0.10	0.07	0.05	0.06	0.12	0.05
1500.00	0.18	0.11	0.08	0.07	0.16	0.06	0.04	0.07
1800.00	0.31	0.10	0.19	0.08	0.37	0.09	0.53	0.05
Tribo filament	0 N		40 N		80 N		120 N	
RPM	RMS	Radius	RMS	Radius	RMS	Radius	RMS	Radius
600.00	0.07	0.07	0.11	0.06	0.02	0.06	0.08	0.06
1000.00	0.22	0.06	0.20	0.05	0.10	0.04	0.11	0.06
1200.00	0.31	0.07	0.61	0.07	0.27	0.07	0.18	0.06
1500.00	0.46	0.09	0.70	0.07	0.34	0.09	0.62	0.07
1800.00	0.61	0.11	0.25	0.07	0.39	0.10	0.28	0.08

The RMS values of vibration acceleration on the bearing housing, shown in Table 1 and illustrated in Figures 3–10. Measuring the RMS values of vibration acceleration is a key tool for monitoring the operational stability and performance of water-lubricated plain bearings, and is also used to diagnose problems and prevent possible failures. A high RMS level of vibration acceleration can signal various types of failure, such as material wear, friction, loosening or structural damage. By analyzing vibration acceleration, it is possible to assess lubrication efficiency. Excessive vibration levels can indicate insufficient lubrication or problems with its even distribution inside the bearing.

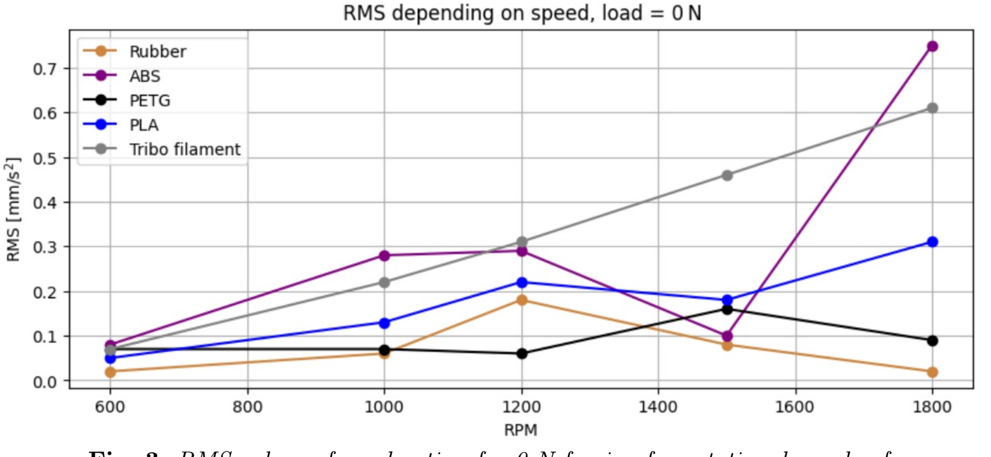

Fig. 3. *RMS values of acceleration for 0 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

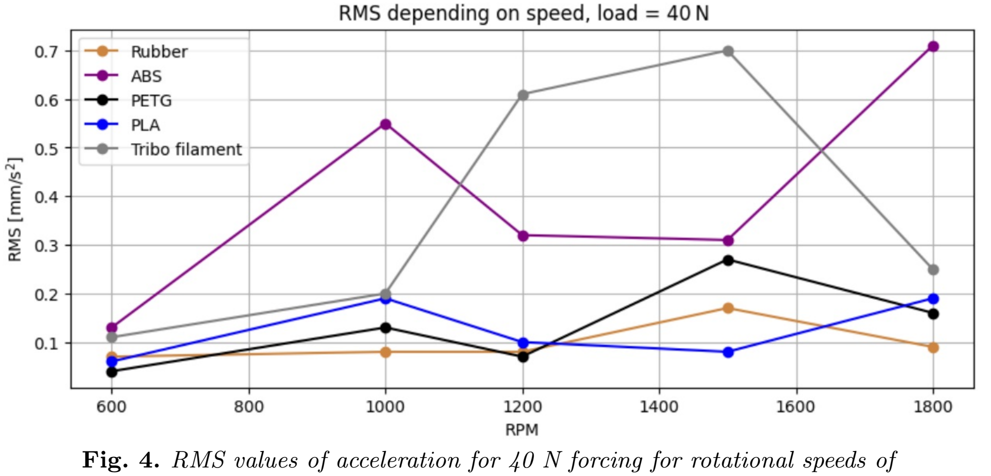

Fig. 4. *RMS values of acceleration for 40 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

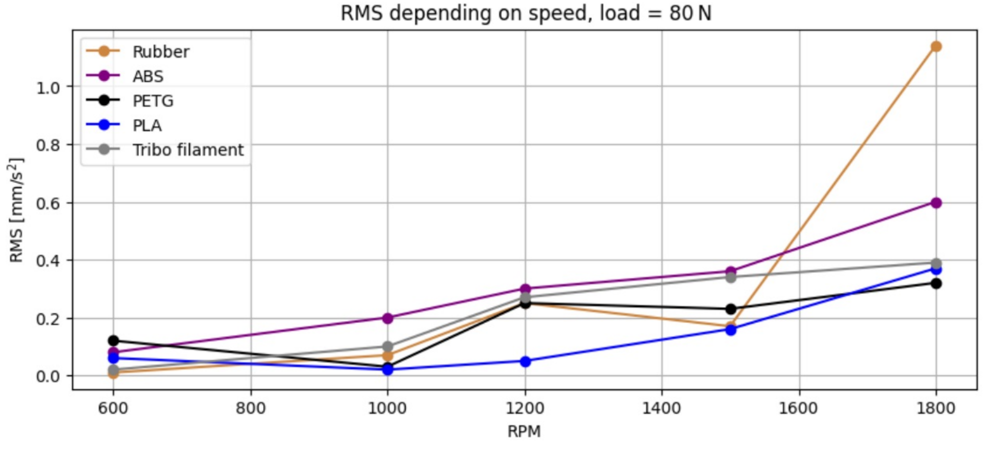

Fig. 5. *RMS values of acceleration for 80 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

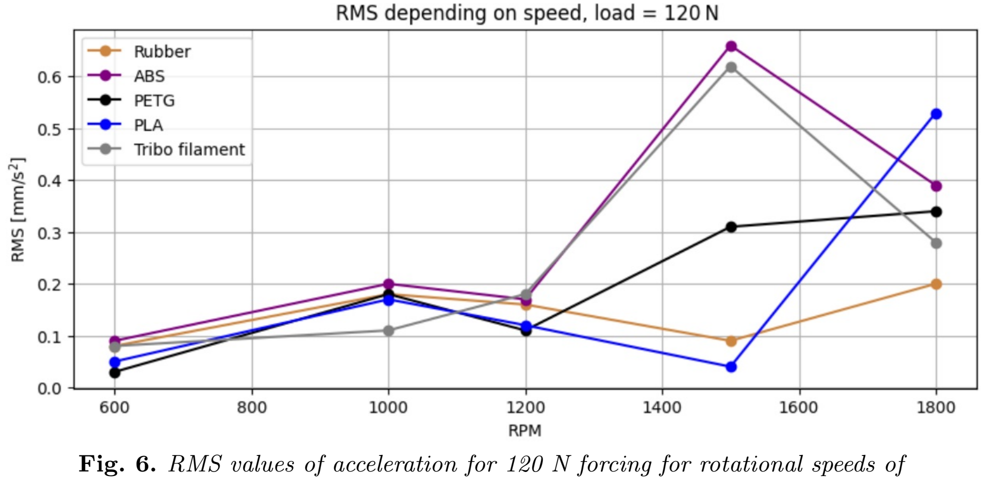

Fig. 6. *RMS values of acceleration for 120 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

Measuring the radius of the trajectory allows the evaluation of a sliding bearing. Changes in this value can indicate problems with achieving proper balance or symmetry in the movement of the shaft. The trajectory radius should remain within the bearing clearance, as its deviation may signify impending wear on the bearing. It can be assumed that the smaller the radius is, the more stable the bearing operates, but in the case of elastohydrodynamic bearings, the acceptable value of the radius is higher. The value of the trajectory radius can change with the load of the bearing. An increase in the radius is associated with overuse or improper operating conditions, which can lead to faster wear of the bearing. The radius of trajectory determines the performance characteristics of the bearing, especially in terms of friction, material wear and stability of shaft motion. Distinct changes in the radius can affect the smoothness of operation and performance of the bearing. Abnormal or unstable values of the trajectory radius can suggest bearing problems such as clearances, excessive wear, friction or mounting errors.

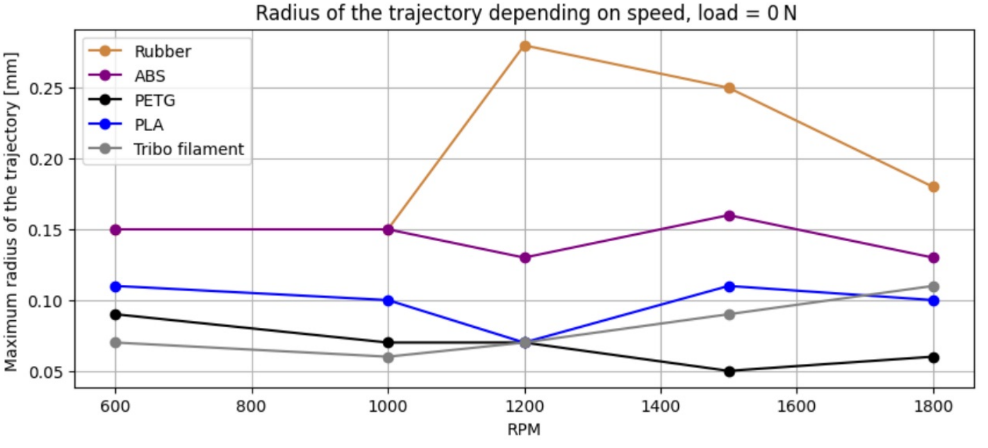

Fig. 7. *Radius of the rotational trajectory for 0 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

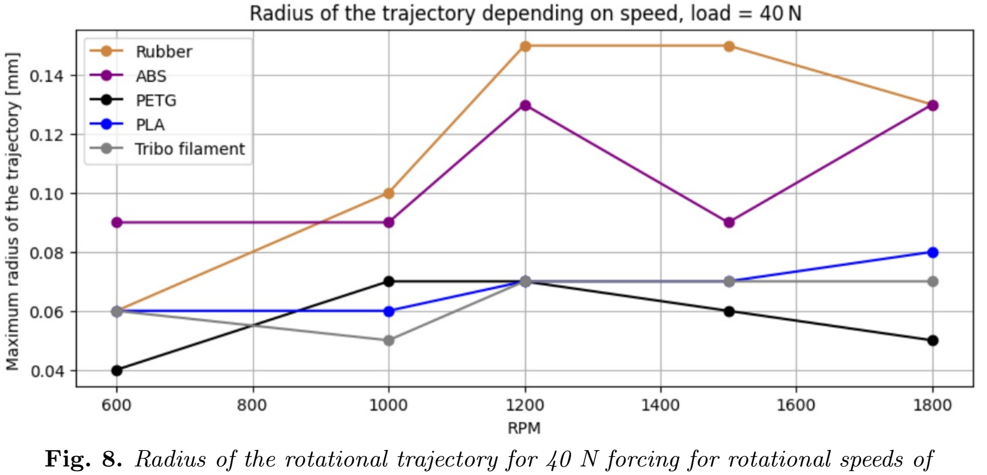

Fig. 8. *Radius of the rotational trajectory for 40 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

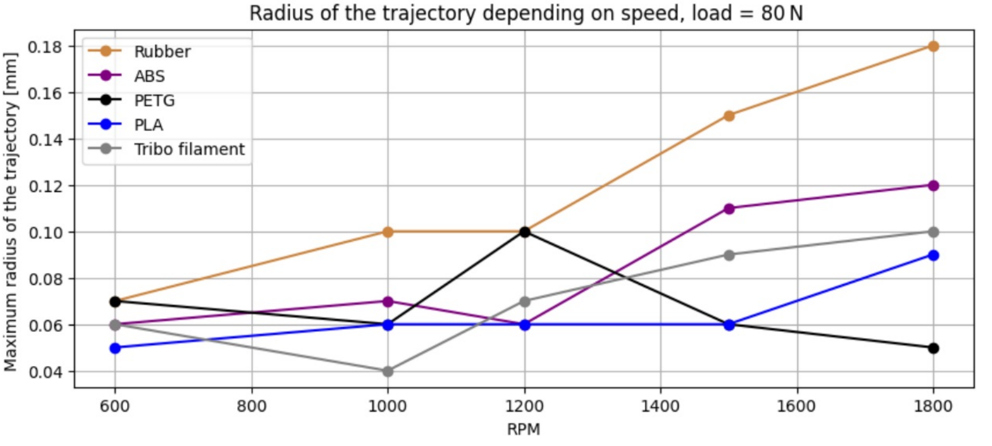

Fig. 9. *Radius of the rotational trajectory for 80 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

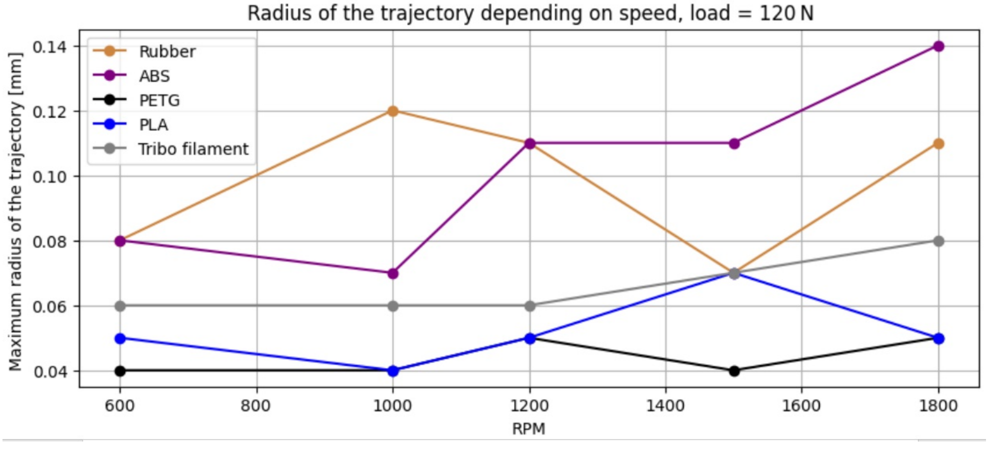

Fig. 10. *Radius of the rotational trajectory for 120 N forcing for rotational speeds of 600 RPM, 1000 RPM, 1200 RPM, 1500 RPM, and 1800 RPM for the tested bearings*

The primary task of a water-lubricated plain bearing is to keep a lubricating wedge that helps reduce frictional forces, water-lubricated bearings have a thinner lubricating wedge compared to conventional bearings lubricated with conventional oils. Another task of the bearing is cooling, water-lubricated bearings have better heat dissipation performance compared to conventional oil-lubricated bearings. Using water as a lubricant instead of oil or mineral grease reduces the negative impact on the environment by eliminating the risk of water and soil pollution. Using water as a lubricant can be simpler and more user-friendly, especially for maintenance and grease replenishment, especially where water is the operating environment. The plain bearings tested showed the best stability of operation at low speeds, according to an analysis of the relationship between speed and average values of machine vibration. In most cases, regardless of the load and type of material, the smallest values of average vibration occurred at rotational speeds of 600 RPM and 1000 RPM. In contrast, increased vibration was often recorded at speeds of 1500 RPM and 1800 RPM.

These results have important implications for the actual application of water-lubricated plain bearings. In practice, this type of bearing most often finds its application in the low-speed area, oscillating between 600 RPM and 1100 RPM. This value indicates optimum operating conditions that are conducive to minimizing vibration and ensuring efficient bearing operation in a water-lubricated environment.

The RMS analysis of acceleration shows that bearings available on the market-made of nitrile rubber has the best vibration damping among the tested bearings within the limits of low loads and speeds. Nevertheless, the radius of the rotational trajectory in the case of bearings made from thermoplastics is consistently smaller, which favors the solutions crafted from thermoplastics. This is because the Young's modulus for rubber is noticeably lower than for thermoplastics at around 0.01 to 0.1 GPa. This means that rubber is more flexible and less rigid than thermoplastics.

The PETG filament bearing has presented promising test results among thermoplastics. The low RMS acceleration values and the smaller radius of the shaft journal trajectory in the bearing than the commercial solution suggest that it is a promising research material worthy of further experimental testing.

The PLA-filament bearing had the best vibration damping properties for higher loads and speeds as can be seen in the RMS plots of vibration velocity for loads of 80 N and 120 N. However, despite these indisputable advantages, the material was not considered for further research due to its biodegradability. Under conditions of temperature 50–60°C and humidity – PLA decomposes in 45–60 days. However, it can be considered for prototype units such as underwater ROVs where missions are relatively short.

Low values of the radius of vibration trajectory are characterized by bearings made of ABS and tribo filament. However, compared to other thermoplastics, the RMS of vibration velocity on the housing is high. Typically, tribo filament has better vibration damping capability. The advantage of tribo filament is that it has a long dry life if the lubricating wedge is lost.

6. CONCLUSIONS

The tested bearings were tested under identical conditions of lateral loads and speeds. Water served as the lubricant, fed through the ferrules, filling the pan completely with the bearing. Analysis showed that the plain bearings tested performed best at lower speeds, typically 600–1100 RPM. This suggests that these bearings achieve the best performance and minimize vibration in this speed range. Studies have shown that bearings made of different materials (such as rubber, PETG, PLA, ABS, Tribo filament) show differences in their vibration damping properties and operational stability. Larger vibration velocities may partly affect the larger radius of the trajectory, which may be related to the function or performance of the bearing. The research conducted, showed on the significant differences of the thermoplastics used on the stability and performance of vibrations transmitted to the pan of water-lubricated plain bearings. The study showed that the stability of plain bearing performance cannot be evaluated only by the RMS of vibration acceleration on the bearing housing or by the trajectory radius. Both of these parameters are important from the user's point of view. For example, for an electric watercraft, a large high RMS vibration acceleration on the housing will indicate vibrations transmitted to the vessel while a large radius will indicate oscillations of the propeller motion which will reduce the efficiency of the ship.

The radius of rotational trajectory of the commercial bearing is in any case larger than the radius of trajectory of bearings made of thermoplastics, which speaks in favor of solutions made of thermoplastics. In most cases, the RMS acceleration values of the commercial bearing are comparable to the design made of PLA and PETG. ABS and tribo-filament bearings fare worse.

In the research work, easily available thermoplastic materials were selected, allowing the shaping of geometry using additive technology; the selected materials should have repeatable operating parameters of the sliding bearing. Further research should examine the performance parameters of bearings made using the additive method from thermoplastics reinforced with glass or carbon fiber, metal powders or crushed wood.

Continuing the analysis and research is essential to enhance our understanding of the mechanisms and optimize the performance of these bearings across various application conditions.

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Identifying Key Factors for Successful Development of Agricultural Biogas Plants in Poland

Grzegorz Ginda*, Marta Szyba*

Abstract. In Poland, the potential for biogas production is high. It is an important source of renewable energy and contributes to reducing methane emissions (a greenhouse gas). In this article, the implementation of an agri-gas plant-construction program was evaluated in individual voivodeships based on materials from the National Agricultural Advisory Center (KOWR), the literature, and statistical data. Based on the collected data, it was concluded that the most meaningful factors for the successful development of Polish agricultural biogas plants were biogas-production technology, substrate availability, energy prices from renewable energy sources, waste-disposal costs, the population density in a commune, and the allocation of places in local spatial-development plans. The DEMATEL technique was used to identify the key developmental factors. The results of the study provide useful information for both governments and local authorities in their searches for effective ways to drive the sector's development.

Keywords: agricultural biogas plants, development, factor, identification, pairwise comparison, DEMATEL

Mathematics Subject Classification: 90B50

JEL Classification: O13, Q59

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1. INTRODUCTION

In act *Polityka Energetyczna Polski do 2040 r. (Poland's Energy Policy until 2040)* (Ministerstwo Energetyki RP, 2019), three goals were formulated; these were "energy security, competitiveness and energy efficiency, and the limited impact of energy on the environment." The last goal is closely related to the development of

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renewable energy sources. It is assumed that, by 2040, this should constitute 28.5% of the share of energy from renewable energy sources (RES) in the gross final energy consumption (and 39.7% in the electricity sector). Renewable energy sources in the power industry are solar energy, wind energy, water energy, biomass, and biogas. Due to limited hydropower resources and difficulties in controlling the supply of wind and solar energy, "the use of biogas will be particularly useful in the combined production of electricity and heat. The advantage is the ability to store energy in biogas, which can be used for regulatory purposes. In terms of general economic use, biogas constitutes additional added value because it enables the management of particularly burdensome waste (e.g., animal waste, landfill gases)" (Ministerstwo Energetyki RP, 2019). The largest biomass resources can be found in agriculture and agri-food processing. These resources are breeding waste (pig and cattle manure, poultry manure), slaughter waste, fruit- and vegetable-processing waste, and distillery waste.

In 2010, the Council of Ministers adopted a document entitled *Kierunki rozwoju biogazowni rolniczych w Polsce w latach 2010–2020 (Directions of development of agricultural biogas plants in Poland in 2010–2020)* (Rada Ministrów RP, 2010). Due to the high costs of the installations that are meant for biogas production, the energy that is potentially produced from it was granted a so-called *blue certificate*, which is more costly than the *green certificates* from other renewable energy sources (e.g., wind farms). The mentioned document assumed that, by 2020, there should be one agri-gas plant in each commune on average (Rada Ministrów RP, 2010). This program will not be implemented because there are only 105 biogas plants that are currently operating; these have the collective capacity to produce approximately 443 mln m^3 of biogas, and the installed capacity of their electricity generators is 109.694 MW (Dyrektor Generalny KOWR, 2019).

In this article, implementations of the agri-gas plant-construction program were evaluated in the individual voivodeships based on materials from the Krajowy Ośrodek Wsparcia Rolnictwa (KOWR) (National Agricultural Advisory Center), the literature, and statistical data. It seems that the existing disproportions were influenced by factors such as the availability of biomass, renewable energy prices, waste-disposal costs, population densities in the voivodeships and communes, and the existence of local spatial development plans in the communes. The DEMATEL (Decision Making Trial and Evaluation Laboratory) method was used to assess the impacts of the factors that were mentioned above on the number of biogas plants; the result of its use was the construction of a diagram that illustrated the strengths of the impacts of the individual factors. An analysis of this diagram may be useful when looking for locations for additional agri-gas plants.

2. AGRICULTURAL BIOGAS

Biogas is produced as a result of the anaerobic fermentation of organic substances under natural conditions (peat bogs, landfills) or in installations that are intended for this purpose. Its basic component is methane, whose contents range from 40–85%. The remaining ingredients are carbon dioxide (in amounts of 16–48%), nitrogen (0.6–7.5%), hydrogen sulfide, and water vapor (Majoch & Jabłońska, 2013). The biogas that is produced in installations is most often burnt in cogeneration units that produce both electricity and heat. Before its combustion in such units, the biogas must be cleaned of its hydrogen sulfide, carbon dioxide, and water vapor.

Because agricultural biogas does not differ in its composition from the biogas from other sources, its name is related to the substrates from which it is produced. In the act *Kierunki rozwoju biogazowni rolniczych w Polsce w latach 2010–2020* (Rada Ministrów RP, 2010), it was defined as: "fuel obtained from the fermentation process of methane from agricultural raw materials, agricultural by-products, liquid or solid animal excrement, by-products or residues from the processing of agricultural, or forest biomass products of origin, excluding the gas that is obtained from the raw materials from sewage-treatment plants and landfills." The substances that were listed in the definition were called substrates.

Agricultural biogas is produced in agri-gas plants (biogas plants) that cover areas of 1–2 ha – the main facilities of which are as follows (Podkówka, 2012):

- storage of solid and liquid substrates,
- pre-mix tank,
- charging hopper,
- fermentation chamber,
- biogas-storage tank,
- biogas-purification device,
- gas-combustion cogeneration unit building,
- control and measurement equipment,
- digestate separation and thickening device.

The biogas that is produced in the fermentation chamber is pumped through pipelines to gas tanks and cogeneration units; most of the time, it is completely burnt on-site in cogeneration units. The generated electric current is sent to the power grid, and the heat is used to heat the substrates (approximately 30%), heating the biogas plant rooms, and selling it (if there are recipients). After being tested for its suitability for fertilizing soils and plants, the digestate is sold as fertilizer, thus providing additional income (Kowalczyk-Juśko, 2014). Digestate is also used in Denmark but not in Norway (Lyng et al., 2020).

The most common substrates in agricultural biogas plants are waste that is harmful to the environment and requires expensive disposal. The disposal of such waste includes slurry, fruit and vegetable residues, distillery stillage, technological sludge from the agri-food industry, re-waste from food processing, and expired food. In 2011, biogas plants processed 469,000 Mg of this waste (including 266,000 Mg of slurry). In 2016, they processed 3,224,000 Mg of waste (including 775,000 Mg of slurry), 665,000 Mg of fruit and vegetable residues, and 476,000 Mg of distillery vinasse. Biogas plants on agricultural farms also used corn silage. In 2011, 109,000 Mg of corn silage was used (23.2% of the mass of all of the substrates); in 2016, this number was 439,000 Mg (13.6%) (Gradziuk, 2017). The data that is quoted shows that, unlike agricultural biogas plants in Germany, Polish agricultural biogas plants use silage to a small extent and, therefore, do not constitute competition for feed production.

Since the waste that is used to produce biogas emits unpleasant odors and the fact that numerous facilities of common agri-gas plants occupy areas of 1–2 ha (Podkówka, 2012), they must be located away from human populations.

3. AGRI-GAS PLANTS IN POLAND

On January 1, 2011, nine biogas plants were entered into the Register of Agricultural Biogas Producers, which is kept by the National Center for Agricultural Support. In total, 14 biogas plants were registered in 2011. The greatest numbers could be found in the following voivodeships: Zachodniopomorskie (5), and Pomorskie (4). A single biogas plant was registered in each of the following voivodeships: Dolnośląskie, Lubelskie, Lubuskie, Śląskie, and Wielkopolskie voivodeships.

Table 1. *Number of agri-gas plants in voivodeships – own study based on (Dyrektor Generalny KOWR, 2019)*

Voivodeship	Population density [people/km^2]	Number of agri-gas plants	As of 26.06.2020
Dolnośląskie	146	10	10
Kujawsko-Pomorskie	116	6	7
Lubelskie	85	7	7
Lubuskie	73	4	4
Łódzkie	137	4	4
Małopolskie	222	2	2
Mazowieckie	150	6	6
Opolskie	106	1	1
Podkarpackie	119	3	3
Podlaskie	59	9	10
Pomorskie	126	9	11
Śląskie	371	2	2
Świętokrzyskie	107	1	1
Warmińsko-Mazurskie	60	10	12
Wielkopolskie	117	11	12
Zachodniopomorskie	75	13	13
Poland	123	–	–

As can be seen in Table 1 posted on page 22, Zachodniopomorskie voivodeship retained its leading position with 13 agricultural biogas plants. The next positions were taken by Wielkopolskie and Warmińsko-Mazurskie voivodeships, which had 12 biogas plants each, followed by Pomorskie Voivodeship (11) and Dolnośląskie and Podlaskie voivodeships (10 each). In turn, Opole and Świętokrzyskie voivodeships had one agricultural biogas plant each, and Śląskie voivodeship had only two.

Analyzing the distribution of the agricultural biogas plants, it can be concluded that most of them were in voivodeships with population densities that were lower than the national average of 123 people per square kilometer; these were the following voivodeships: Warmińsko-Mazurskie (60), Zachodniopomorskie (75), Podlaskie (59), and Wielkopolskie (11). The existence of a large number of biogas plants in Dolnośląskie, Pomorskie, and Mazowieckie voivodeships can be explained by the existence of several large urban centers, while the populations were lower in the rural and urban-rural communes. This is illustrated by the data in Table 2, which shows that, as of March 5, 2019, 50.5% of the Polish agri-gas plants were located in communes with population densities of less than 50 people per square kilometer. The percentage as of June 2, 2020, this share increased to 53.3%. During the period that was analyzed, the number of agri-gas plants that were located in communes with population densities within a range of 25.1–50 people per square kilometer increased the most (by six percentage points).

Table 2. *Number of agri-gas plants vs. population density in communes – own study based on (Dyrektor Generalny KOWR, 2019)*

Population density [people/km^2]	**Number of agri-gas plants**	**As of 26.06.2020**
< 25.0	12	13
25.1–50.0	37	43
50.1–75.0	19	18
75.0–100.0	13	15
100.1–125.0	6	5
125.1–150.0	4	4
> 150.1	6	7
Overall	97	105

The numbers of new agricultural biogas producers that were registered during the years of 2011–2020 are presented in Table 3. The highest numbers (21 each) were in 2015 and 2016, while the lowest were in 2018. The collapse of the growth trend after 2016 cannot be associated with the profitability of the operations of agri-gas plants, as *blue certificates* had been in force for the energy that is produced in agricultural biogas plants since 2016 – the price of which being 150 to 200% greater than the prices of the *green certificates* for the other renewable energy sources (Iwaszczuk et al., 2019).

Table 3. *Number of registered agri-gas plants for period of 2011–2020 – own study based on (Dyrektor Generalny KOWR, 2019)*

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020[a]
Number	14	10	13	16	21	21	3	1	7	5

[a]through 26.06.2020

4. FACTORS THAT INFLUENCE LOCATIONS OF AGRICULTURAL BIOGAS PLANTS IN POLAND

Large resources of the substrates that are produced by the Polish agri-food sector are used to a small extent for the production of agricultural biogas, which is associated with the threats that are related to the operation of agri-gas plants. As recently as 2015, Igliński and his colleagues conducted a SWOT analysis; as a result of this, they identified the most important threats as follows (Igliński et al., 2015):

- instability of prices of agricultural substrates,
- no guarantee of stable input supplies,
- decrease in prices of conventional fuels.

The SWOT analysis that was conducted by Iwaszczuk et al. (2019) listed the following threats:

- price drop of blue certificates,
- instability of prices of substrates from crops for energy purposes,
- decrease in prices for disposal of agri-food waste,
- decrease in prices of conventional fuels,
- increase in land and real estate taxes,
- closure of large agri-food processing plant that supplied substrates to biogas plant,
- epidemic among animals, causing destruction of entire herds,
- recurring natural disasters.

The weaknesses in both analyses included the resistance of the local community and the long investment process, which can be associated with both this resistance and the lack of local spatial-development plans in most commune areas.

In order to explain the reasons for the small number of biogas plants in Poland, Igliński et al. (2020) used a PEST analysis, which takes macroenvironmental factors into account: political (P), economic (E), social (S), and technological (T). The analysis showed that the greatest threats to the development of agricultural biogas plants were the strong conventional energy lobby, an unfriendly energy policy, an uncertain global economic situation, the low possibilities of financing biogas investments from investors' own funds, the low social acceptance of biogas technology, the poor condition of the power grid in Poland, and the poor cooperation between industry and science.

After analyzing the literature on the issue of agricultural biogas production, the statistical data, and interviews in those towns where agri-gas plants operated (Piekoszów, Liszkowo), the authors decided to investigate the factors that supported the construction of agricultural biogas plants. Such factors were considered to be as follows:

- knowledge about process, which affects safety and eliminates operational nuisances (W),
- availability of substrates (price, transport costs, regularity of deliveries) (S),

– price of energy from renewable energy sources, ensuring economic profitability of operation (C),
– costs of agri-food waste disposal and possibility of other uses (U),
– population density in commune (G),
– spatial order in commune that resulted from local development plan or historically shaped residential development (P).

5. FACTORS

During the analysis, the following $n = 6$ factors were taken into account:

1) knowledge (W), understood in context of nuisance to environment,
2) substrates (S) – waste or corn cultivation,
3) energy price (C), including certificates for electricity and heat,
4) cost of waste disposal (U), related to possibility of using it for purposes other than biogas production,
5) population density in commune (G),
6) adopted local development plan in commune (P) – urban order related to local history.

6. DIRECT INFLUENCE OF FACTORS

During the analysis of the influence of the factors, original DEMATEL version was used; this allowed us to express the strength of the direct influence of one of the compared factors on another factor using the following scale:

0. no direct influence of first of pair of compared factors on second,
1. little influence of first factor,
2. high influence,
3. very high influence,
4. extreme influence.

The set of n^2 estimates of the direct impact of the factors expresses the structure of their direct impact. It is worth noting that, when determining the structure of direct influence:

– we take the possibility of both directions of the interactions that occur between the i-th and j-th factors into account $(i, j = 1 \dots n)$;
– it is not possible for an individual factor to have a direct influence on itself.

In the case of the set of factors { W, S, C, U, G, P }, the assumed structure of the direct influence is illustrated by a directed graph – *direct influence graph* (presented in Figure 1). The lack of a direct influence of the factors corresponds to the lack of an arc that connects the vertices of the factors. However, the direct influence at the level of the individual scale degrees is expressed by different types of arc lines:

– dotted line corresponds to direct impact assessment of Level 1,
– dash line – Level 2,
– thin solid line – Level 3,
– bold solid line – Level 4.

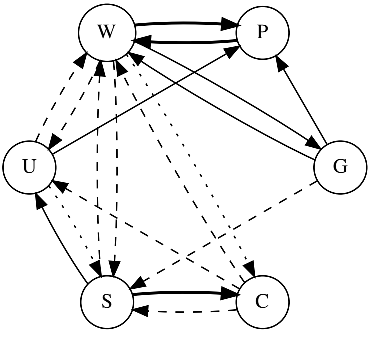

Fig. 1. *Assumed structure of direct influence of factors*

Note that the image of the structure of the direct influence may also suggest the hierarchical nature of the structure of direct influence – as, e.g., in Figure 2.

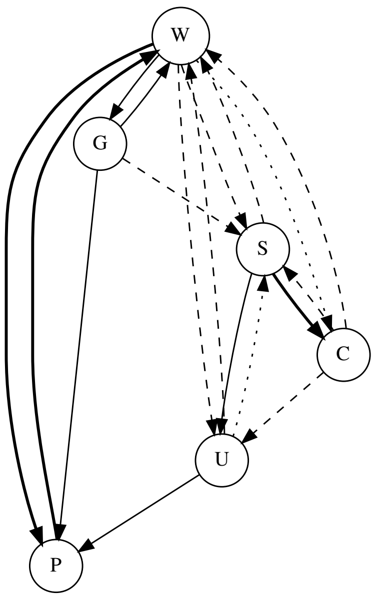

Fig. 2. *Suggested hierarchy of direct influence factors structure*

The structure of the direct influence is also expressed in a dedicated square *matrix of direct influence* X^* with n rows and n columns that correspond to the subsequent factors. In the case under consideration, assume that the order of factors W, S, C, U, G, and P takes the following form:

$$X^* = \begin{bmatrix} 0 & 2 & 1 & 2 & 3 & 4 \\ 2 & 0 & 4 & 3 & 0 & 0 \\ 2 & 2 & 0 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 3 \\ 3 & 2 & 0 & 0 & 0 & 3 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{1}$$

As a result of dividing it by the maximum row sum of its elements (which is $\lambda = 12$), we obtain its normalized form:

$$X = \frac{X^*}{\lambda}, \tag{2}$$

which should meet the following condition:

$$\lim_{k \to \infty} X^k = 0. \tag{3}$$

Based on this, we finally obtain the *total influence structure*, expressed by *total influence matrix* T:

$$T = X\,(I - X)^{-1}. \tag{4}$$

It is worth noting that the total impact can also be expressed as the sum of the impact:

- direct X (2);
- indirect impact resulting from transmission, which – thanks to Property 3 – can be estimated using following formula:

$$\mathit{\Delta} X = X^2\,(I - X)^{-1}. \tag{5}$$

Thus:

$$T = X + \mathit{\Delta} X. \tag{6}$$

By applying both Formulas (4) and (6), we finally obtain the following form of the total impact structure:

$$T = \begin{bmatrix} 0.4938 & 0.3857 & 0.2530 & 0.3876 & 0.3735 & 0.6882 \\ 0.4854 & 0.2135 & 0.4449 & 0.4584 & 0.1213 & 0.3067 \\ 0.3989 & 0.2995 & 0.1331 & 0.3302 & 0.0997 & 0.2404 \\ 0.4139 & 0.1975 & 0.1003 & 0.1351 & 0.1035 & 0.4476 \\ 0.5788 & 0.3308 & 0.1585 & 0.2056 & 0.1447 & 0.5305 \\ 0.4979 & 0.1286 & 0.0843 & 0.1292 & 0.1245 & 0.2294 \end{bmatrix}, \tag{7}$$

where:

$$\mathit{\Delta} X = \begin{bmatrix} 0.4938 & 0.2190 & 0.1697 & 0.2209 & 0.1235 & 0.3549 \\ 0.3187 & 0.2135 & 0.1116 & 0.2084 & 0.1213 & 0.3067 \\ 0.2322 & 0.1328 & 0.1331 & 0.1635 & 0.0997 & 0.2404 \\ 0.2472 & 0.1142 & 0.1003 & 0.1351 & 0.1035 & 0.1976 \\ 0.3288 & 0.1641 & 0.1585 & 0.2056 & 0.1447 & 0.2805 \\ 0.1646 & 0.1286 & 0.0843 & 0.1292 & 0.1245 & 0.2294 \end{bmatrix}. \tag{8}$$

Based on the obtained structure of the total influence T (Formula (7)), a pair of indicators can be obtained for each of the factors:

$$\begin{aligned} \forall_{i=1\ldots n} \ s_i^+ &= \sum_j^n t_{ij} + t_{ji}, \\ \forall_{i=1\ldots n} \ s_i^- &= \sum_j^n t_{ij} - t_{ji}, \end{aligned} \tag{9}$$

which are called indicators, respectively: position or prominence (en.*prominence*) s_i^+ and relation (en. *relation*) s_i^-. The first expresses the strength of the connections of the i-th factor with the factors[1]. The second one allows us to express the character – causal ($s_i^- > 0$), consequential ($s_i^- < 0$), or neutral ($s_i^- = 0$) – the i-th next factor. Therefore, the relationship and position indicators help in the two-dimensional classification of the factors, which are carried out according to their nature and the strength of the connections between them. The results of such a classification are illustrated in Table 4.

Table 4. *Two-dimensional factor classification*

Factor:	**W**	**S**	**C**	**U**	**G**	**P**
i	1	2	3	4	5	6
s_i^+	5,450	3,585	2,675	3,044	2,916	3,636
s_i^-	−0.287	+0.474	+0.327	−0.248	+0.981	−1.249
Connections	Strong	Medium	Weak	Weak	Weak	Medium
Nature	Effect	Cause	Cause	Effect	Cause	Effect
Quarter	IV	II	II	III	II	III

Note that the term *quadrants* refers to the quadrants of the $s^+ - s^-$ coordinate system, created by shifting the ordinate axes s^- to point $s^+ = 4.063$, located on the abscissa of s^+, in the middle of the interval between the upper ($s_1^+ = 5.450$) and lower ($s^+ = 2.675$) limit prominence index (compare: Figure 3).

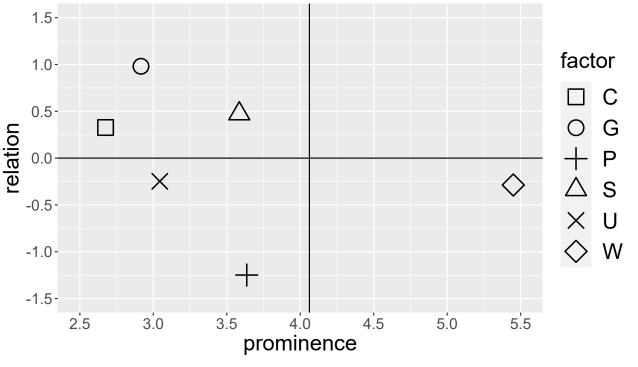

Fig. 3. *Graphical illustration of factor classification results*

[1] Due to the possible indirect influence of the other factors – also with themselves!

Based on the content of Table 4 it can be concluded that:

1) The factor with the most clear causal character is factor G, which is rather a weakly related factor. Factor C is also similar in nature. The list of causes is exhausted by factor S, which distinguishes it from the rest of the causes with rather average connections.
2) The most visible effect is the P factor, which stands out from the rest of the effects due to its rather average level of connections (the U factor has weak connections and the W factor has strong connections).

The analysis shows that the key factors for the development of biogas plants in Poland include population density G, energy price C and availability of substrates S.

7. DISCUSSION OF THE RESULTS OF THE ANALYSIS USING THE PEST AND DEMATEL METHODS

In the PEST method, Igliński et al. (2020) presented the examined factors on a point scale in which a highly unfavorable factor was assigned a value of 1 and a very favorable factor a value of 5. Based on the data that was obtained in our surveys, it was assumed that, in the areas of the political, social, and technical environments, a very favorable factor (value 5.0) was Poland's membership in the EU. The same value in the area of economic environment was adopted for globalization, as it ensures the free flow of goods, capital, and services. Only the conventional energy lobby in the political environment was considered to be a highly unfavorable factor (value 1). Unfavorable factors (value 2) were assumed for the renewable energy policy in the area of the political environment and the ability of science and economy to cooperate in the area of the technical environment. The average value of the selected factors was calculated for each environment. When assessing the environment, it was assumed that the environment was highly unfavorable if the mean was less than 2.0; unfavorable when it was within a range of 2.00–2.99; neutral (range 3.00–3.49); favorable for a mean within a range of 3.50–4.49, and very favorable for a mean greater than 4.50. The following average values were obtained for the individual areas:

– political environment: 3.25,
– economic environment: 3.88,
– social environment: 3.56,
– technical environment: 3.25.

The values that are presented above show that the reason for the slow development of agricultural biogas plants is the lack of areas with very favorable conditions. Only the economic environment can be considered to be favorable, while the rest are neutral.

The analysis using the DEMATEL method showed that the key factors for the locations of agricultural biogas plants were population density, energy prices, and the availability of substrates.

8. CONCLUSIONS

The development of biogas plants in Poland should be based on modern management, which would use modern decision-support methods. The article presents the practical application of the DEMATEL method, which was used to determine the matrix of the mutual connections between the influences of each pair of the factors (knowledge, substrates, energy price, cost of waste disposal, population density, and the adopted local development plan in the commune). A total influence matrix T was constructed, and a pair of indicators (called the position and relationship indicators) were obtained for each factor. The factors that had the greatest and least influence on the development of biogas plants in Poland were determined.

As a result of the analysis that was carried out regarding the development of the biogas plant market in Poland, the most important factors included the following:

– Population density G – this result was not accidental, as the operation of an agricultural biogas plant has a strong impact on the people who live in its vicinity; this mainly concerns the odors that are released from substrate storage containers in a biogas plant and the odors that are released from the anaerobic fermentation chambers.
– Price of energy C – this depends on the price of "blue certificates," which were introduced in Poland in 2016; the incentive for investors was the higher price of "blue certificates" as compared to "green" certificates. Setting the price at a higher level was intended to compensate for the high costs of investing in agri-gas plants.
– Availability of S substrates – the production and quality of agricultural biogas depends on the type of substrate that is used in the fermentation process; most often, biogas is produced from farm animal excrement and corn stalks. Another source of substrates may be agri-food and slaughterhouse waste – the use of which allows for the production of very good quality biogas, which is characterized by a high methane content. After compression, such a biogas can be used to power automobile engines.

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Risk Management in Heating Industry

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Abstract. The heating industry plays a very important role in countries with cold and temperate climates; together with the power industry, it determines the energy security of these countries. The aim of this article is to examine the risks that threaten the stability of heating companies. The study is based on an analysis of the scientific literature and the macroeconomic environment as well as on interviews with employees from the heating industry. The article identifies the risks and divides them into three groups: general economic, industry, and specific risks. A risk map was drawn, and a qualitative analysis of their impact was carried out for the aforementioned companies. This map is a useful tool for making decisions that are related to risk management and ensuring the stability of the functioning of business entities as well as gaining information for both government and local authorities in the search for effective ways to drive the sector's development.

Keywords: energetic safety, district heating, risk-management strategies, decision-making, economic stability

Mathematics Subject Classification: 91B05

JEL Classification: G32, D81

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1. INTRODUCTION AND METHODOLOGY

The globalization of the world economy has led to the integration of national economies; this makes it possible to not only import goods and services but also penetrate various threats – the number and range of which are constantly growing. The problem of risk is of a particular importance in the case of economic activity. It is known that the success of a business depends on the correctness of the chosen business strategy as well as taking the possibility of hazardous situations into account. It would be naive to think that business is possible without risk; a businessman must take the impact of many threats (risks) into account on any implemented projects (e.g., investments, projects) and determine the probability of their occurrence and the degree of their impact. Such activities are called risk management.

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Risk management must be long-term and consistent with an overall corporate management strategy. To build an effective risk-management strategy, managers need to answer many questions, including which risk-management strategy to adopt, which strategy will be the best under the given market conditions, and whether the adopted strategy will be effective in a dynamically changing environment.

Decision-makers can rely on both their own knowledge and experience as well as on external experts or those companies that advise on risk management. Regardless of who creates the strategy, it must be adapted to an organization's purpose, mission, and vision, its risk appetite, and its attitudes toward risk. Furthermore, modern risk management must be very dynamic. The efficiency of its operation largely depends on the speed of the response to changes in market conditions, the phases of the business cycle, the company's financial condition, and many other external and internal factors.

Many scientific publications have been devoted to research in heat engineering. For example, Edling and Danks (2021) focused their attention on state policies and private programs in the development of local and renewable heating technologies. Schüwer and Schneider (2018) analyzed the possibilities of electrifying industrial-process heat. Nielsen and her coauthors investigated unconventional excess heat sources for district heating on examples in Germany, Spain, and France (Nielsen et al., 2020). Egging-Bratseth et al. (2021) considered the possibilities of seasonal storage and demand management in district-heating networks under conditions of changing demand. Lygnerud and Werner (2018) investigated the risk of industrial excess-heat recovery in district-heating systems. They considered that the recovery of excess industrial heat for use in district-heating systems can be characterized by great political interest, high potential, low use, and often high profitability. These characteristics reveal that there are barriers that inhibit its greater use; an identified barrier is the risk that industries with excess heat can terminate their activities, resulting in a loss of heat recovery (Lygnerud & Werner, 2018).

There are many publications that describe the connection of traditional district-heating networks with renewable energy sources (Sorknæs et al., 2020), including heat pumps (Mirl et al., 2018; Wei et al., 2021) and geothermal and solar resources (Chen et al., 2020). There are also publications that are devoted to analyzing heating systems in countries (Triebs et al., 2021) or regions (Zhang et al., 2021). However, it is difficult to find publications in which the authors would comprehensively analyze the risk of heating companies' operations.

The purpose of the article is to identify and analyze the risks in the activities of a heating company; these are broken down into three groups: general economic, industry, and specific risks. Three methods were used to achieve this goal: 1) an analysis of the literature and a legal act analysis, 2) an analysis of the macroeconomic environment, and 3) interviews with employees from the heating industry.

2. IDENTIFICATION OF GENERAL ECONOMIC RISK FACTORS

General economic risks threaten all enterprises that operate in the country. The most important of them include the risk of an increase in interest rates, the risk of fluctuations in the exchange rates, and the risk of force majeure.

Interest-rate risk is related to the high sensitivity of financial markets, which translates into fluctuations in the price of money (interest rate) as well as the value of assets and liabilities. Heating companies rarely invest in financial assets (such as, e.g., stocks or bonds), so this part of the risk can be ignored. However, they often use borrowed capital (liabilities) to finance their activities. Borrowed capital may come from the following:

– bank loan,
– issue and sale of debt securities (bonds, promissory notes, commercial papers).

The interest rate on bank loans is usually floating and calculated according to the market interest rate – the amount of which depends on the demand and supply of money at a given time. In turn, money demand and supply depend on many external factors. In Poland, the interest on loans is most often calculated according to the six-month WIBOR 6M (Warsaw Interbank Offer Rate).

As shown in Figure 1, this rate does not change much over a short period time – especially during periods of economic stagnation. On the other hand, the WIBOR rate can increase significantly during periods of changes in the phases of the business cycle, resulting in increases in loan costs.

Fig. 1. *WIBOR 6M (vertical axis) during period of January 1, 2001, through August 1, 2021 – source: Bankier.pl, 2021*

The same is true for taking out a bond loan. For a bond issuer, the riskiest is a fixed-interest rate. Then, the opportunity costs (the costs of debt service) increase with a falling trend in the interest-rate market. With a floating interest rate along with a decrease in interest rates, however, the cost of debt servicing also decreases. This is why it is so important to build a balanced loan portfolio – it must be diversified in terms of its type (loans and debt securities), term (different periods), and type of interest rate (fixed and variable). Interest-rate risk can also be managed by purchasing derivatives for deposits and bonds as well as by controlling the structures of assets and liabilities.

There are various derivatives that can be used to manage this risk, such as interest-rate swaps, currency interest-rate swaps, options, forward contracts, and futures contracts. The type of interest rate on loans should also be approached with caution. Under the conditions of an increase in the market interest rate, a fixed rate is more profitable; in the case of a downward trend, a variable rate is best.

Exchange-rate risk. The activity of heating companies usually has a regional scope; therefore, it is unlikely that the risk of exchange rates that are related to heat exports will be a threat to them. However, these companies can buy imported fuels (e.g., natural gas or coal).

During the years of 2002–2019, the share of coal fuels in Poland decreased by 10.7%, while the share of gas fuels increased by 5.8% and that of renewable energy sources (RES) increased by 6.6%. In 2019, the heating sector consumed 32,151,990.2 GJ of high-methane gas (which represented 7.54% of the consumption of all of the fuels for heat production), 8,215,813.5 GJ of nitrogen-rich gas (93%), 150,469.5 GJ of biogas (0.04%), and 838,423.7 GJ of other RES (0.20% of consumption). However, the greatest amount of heat was produced with the use of coal fuels – 71.0% (Urząd Regulacji Energetyki, 2022, pp. 13–14). The coal came from both Poland and abroad, as the country has been a net importer of hard coal from beyond its eastern border since 2008 (the settlement currency is most often the US dollar). The largest heating companies balance the purchase of domestic raw materials with imported raw materials; therefore, the risk of the transaction exchange rate increases year by year. For example, coal imports to Poland increased by 60% in 2017 (to 13.3 million tons of coal) according to the Business Insider Polska portal; of this, more than half of the raw material (8.7 million tons) came from Russia (Biznes Insider, 2018).

According to Eurostat data, 8.86 million tons of coal came to Poland after three quarters of 2020 (including 6.52 million tons from Russia). The more than 73% share of Russian coal during this period was the highest in years. For a comparison, this amounted to 67% in 2019 (raw material imports amounted to 11.97 million Mg after nine months, including 8.02 million Mg from Russia). In turn, up to 13.65 million Mg came from abroad during the period of January–September 2018 (including 9.9 million Mg from Russia, which accounted for 72.5% of all coal supplies) (Baca-Pogorzelska, 2021). The year 2018 was a record-breaking year for coal imports to Poland.

The data that was mentioned above shows that heating companies must take the risk of exchange rates into account – especially the risk of an increase in the currency exchange rate of a contract settlement. In addition, there are other threats: the risk of an increase in the fuel price, the risk of its transportation costs (external risks), and the risk of losing financial liquidity (internal risk).

The risk of force majeure is associated with the effects of natural (floods, hurricanes, fires, earthquakes, tornadoes) and human factors (armed conflicts, acts of sabotage, general strikes). The result of these actions may be a temporary suspension of the provision of services and catastrophic damage, completely preventing business activity. An enterprise has no influence on such phenomena; only insurance indemnities can facilitate the process of rebuilding fixed assets.

Identifying and analyzing business risk is an element that determines the possibility of creating enterprise value. This requires one to develop an action strategy to achieve the basic goals of one's economic activity.

3. IDENTIFICATION OF INDUSTRY RISK FACTORS

Industry risk is a risk that is typical for the enterprises the operate in a given industry. The typical risk for the heating industry includes price risk (changes in the prices of raw energy materials and heat prices), the risk of a decrease in demand for heat, competition risk, regulatory and legal risk, and environmental risk.

The risk of changes in the price of energy resources. Thermal energy can be obtained from various sources, ranging from coal and gas to biomass and waste. However, the heating industry in Poland is mainly based on coal.

The increase in the price of coal is influenced by the decrease in its production in domestic mines and the increase in the costs of its exploitation (owing to the need to descend to lower and lower coal seams). Moreover, the prices of energy commodities have become world prices as a result of the globalization of the world economy, and they shape the spot and futures markets. Their variability depends on many factors that are very difficult to identify, as not all are measurable.

For example, raw material prices are influenced by various events such as a change in a political cabinet or the directions of the economic development of the largest coal-producing countries, increased demand from dynamically developing countries (such as China), the collegial and individual decisions of the major fuel suppliers in the world, wars, strikes, earthquakes, volcanic eruptions, and other natural disasters. The economic trends that can be observed on global markets give an impulse for similar phenomena on the domestic coal market and also constitute a reference price for the contracts that have been concluded in Poland.

The increase in fuel prices in the world markets increases the costs of heat production while reducing the revenues of a heating company. If an increase in costs cannot be covered by increasing the price of heat (tariffs are set by Urząd Regulacji Energetyki (URE) (Energy Regulatory Office), the company will face financial risk and, consequently, the risk of losing liquidity.

The risk of changes in the price of heat. Heating companies are characterized by locality: heat is not transmitted over long distances (mainly for economic reasons). Each region is most often served by a heating plant that is the only heat supplier in the area (the so-called natural monopoly). For this reason, the heat market is regulated by the URE, which imposes a method for calculating the heat price according to the prescribed rules. According to the regulation, the price of heat should cover the costs of heat generation, its trading, and its transmission as well as the return on capital that is involved in the activities of entities in the heating industry (*Rozporządzenie Ministra Klimatu z dnia 7 kwietnia 2020 r. w sprawie szczegółowych zasad kształtowania i kalkulacji taryf oraz rozliczeń z tytułu zaopatrzenia w ciepło*, 2020).

This method of calculating the price is intended to motivate companies to invest (in order to modernize and increase operational efficiency), the costs of which will be covered by increasing the price tariff. The regulation of heat prices, however, does not take sudden changes in market fuel prices, transmission losses, weather risk, and other unexpected changes in the external environment into account.

The risk of falling demand for district heat. There can be many reasons for falling demand for heat, including modern energy-saving trends: the thermo-modernization of buildings, the installation of weather automation in heating nodes, and the installation of heat regulators in rooms.

Currently, many owners of single-family houses install renewable energy devices; e.g., photovoltaic panels, solar panels, and heat pumps. These are used to produce electricity and heat. Public building managers are also increasingly installing such devices because they are often supported by the state. However, for heating companies, reducing the number of users means lowering the revenues on which they originally relied upon when implementing their investment.

In addition, some residents consciously reduce the consumption of district heat due to a worsening economic situation. Another reason is the migration of people to large cities (mainly for work), which has contributed to the depopulation of smaller towns. As a result, the demand for heat has also fallen in such smaller municipalities.

Competition risk. Generally, heating companies have their own heat-sales areas and an infrastructure that is adapted to them. The entry of other entities into the same area will not be profitable; therefore, there is no competition from similar business entities in this industry. However, competition may be created by dispersed heat sources and local gas boiler houses, (especially those operating on the biomethane that is produced in biogas plants – the number of which is constantly growing in Poland). These are produced in landfills, sewage-treatment plants, and in the countrysides near farms.

Regulatory and legal risk results from changes in the economic policy of the state. They translate into restrictions of the freedom of economic activity in the forms of granting concessions, conducting constant supervision over the activity, changing the existing law, and introducing new regulations.

In the case of heating, this risk manifests itself mainly in the form of tariffs for heat and the income of energy companies. Additionally, environmental policy (increasingly stringent requirements for greenhouse gas emissions) affects their financial situations due to the need to invest in additional purification filters to capture the by-products from fuel combustion. This risk has an even greater impact on combined heat and power plants, which must comply with the requirements for both activities. In the event that the modernization or installation of modern heating devices that capture emitted pollutants is not feasible, a company will be required to pay high financial penalties or purchase additional emission allowances.

The environmental risk is related to the previous type of risk, because changes in laws and regulations require taking certain actions to meet them. This is connected with the necessity of the deep modernization of heating plants (or combined heat and power plants) or the installation of new devices for the treatment of post-production waste (mainly the gases and dust that are emitted into the atmosphere). In the case of ecological coal combustion, a large one-off expenditure is required for modernization (e.g., for the construction of a wet flue gas desulfurization and denitrification installation, which can reduce sulfur oxide and dust emissions five-fold and nitrogen oxide emissions three-fold). In the case of natural gas, such installations do not need to be installed, as this is considered to be the cleanest of all of the fossil fuels. The same is true for biomethane.

4. IDENTIFICATION OF SPECIFIC RISK FACTORS

Each company is characterized by an individual set of dangers (specific risks). In the case of the heating industry, the group of specific risks includes the risk of disruptions in fuel supplies, the risk of inadequate fuel quality, technological defects, credit risks, and weather conditions that reduce the heat consumption by users. They also overlap with some types of internal risks (e.g., technological risk).

Risk of fuel supply. The continuity of fuel supplies and their quality determine the stability of heating companies. The Polish heating sector is mainly based on coal fuels, and not much will change in the near future because most of the gas and heating oil (obtained from crude oil) are imported. There is also no possibility of a rapid increase in the share of renewable energy sources, which amounted to approximately 10% of the total consumption in 2019 (Urząd Regulacji Energetyki, 2022, pp. 13–14). Therefore, the focus here will only be on coal.

As in any other business activity, the continuity of supply is a very important issue that can be disrupted by many different factors, from supplier oversight through logistic and atmospheric problems to failures in coal mines and natural disasters. A company should be prepared for such situations by maintaining an emergency coal reserve.

However, the size of such a reserve is unknown; a too-small reserve may cause problems with the continuity of heat production (which is unacceptable), and a too-large reserve may generate additional costs of storing and securing the fuel. Similar threats may also occur in the cases of the supplies of heating oil, biomass, and other fuels (with the exception of gas, whose storage is nearly impossible).

Fuel-quality risk. Another risk may be related to the quality of coal that does not correspond to the contracted parameters. The quality of coal depends on its calorific value and the contents of the admixtures (including ash and combustible sulfur). Low qualities of coal are associated with increases in the emissions of pollutants and the additional costs of their management.

Technological risk includes threats that are related to downtimes as well as failures in the operations of devices. Such events can occur at very low air temperatures when the pipes and valves in district-heating networks burst. Interruptions in heating supplies can lead to the destruction of internal heating systems in buildings, which will likely result in the filing of civil lawsuits against a heating company. Due to the unique role of heating companies, this is one of the most important risks for which one must be prepared.

In this case, insurance will not help; the only solution is to eliminate the cause of the failure as soon as possible. This is why it is so important to modernize old heating networks. To ensure their stable operation (including the maintenance of pressure in networks, the flow through boilers, and the operation of exhaust fans), it is essential to provide automatic backup external power-supply systems and own generators. In the event of interruptions in heat supplies, clear procedures to reduce supplies are important, taking special-purpose buildings into account.

Credit risk. Sources of credit risk are loans and credit; banks and other financial institutions are the most vulnerable to these. In general, this risk can be defined as the threat of the borrower's failure to meet the terms of a loan agreement. So, how can a heating company be associated with a given type of risk?

Increasing the prices of heat (which is beneficial for the heating sector) may expose the company to credit risk, as the risk of reducing receipts from customer fees (due to their financial problems) increases. In this situation, the heating company may grant them a trade credit.

Weather risk. The heating industry is one of the few industries that are sensitive to weather conditions – especially to air temperature and wind force; this is defined as sensitivity to non-catastrophic weather risk. In this industry, this is associated with reductions of heat consumption during periods of unusually high temperatures during the heating season. The strength of the wind is also important because, with strong winds, heat consumption is higher (and vice versa). Weather risk can be classified as both industry and specific risks; it depends on whether the climatic conditions in the country are the same.

Although the climate in Poland is relatively the same (temperate climate, heat group, transitional type), the following reasons speak in favor of classifying this risk as being specific:

– heterogeneity of terrain (even in same voivodeship, there are lowland and mountainous areas — lower temperatures are most often observed in mountainous areas),
– in Poland, some regions are usually warmer (south and southwest),
– other regions of Poland are colder (north and northeast),
– Baltic Sea region is characterized by stronger winds,
– it happens that temperatures in Poland differ by several degrees at same time.

Currently, the risk of non-catastrophic weather is the result of global climate change; this results in extremely high summer temperatures and mild weather conditions during the heating season. The latter aspect is alarming among the management staffs in heating companies.

Specific risk also includes all types of threats that are related to internal factors in company operations, such as inadequate employee qualifications, low work efficiency, improper work organization, poor management structures, a lack of appropriate control mechanisms, and the improper operations of machines and devices.

5. RISK MAP FOR HEATING COMPANIES

External and internal risk factors can be related to each other, neutralizing or increasing the risks. Therefore, an integrated approach for building a diversified risk-management strategy is important. Table 1 presents the different types of risk (divided into three groups), with attachments to their threats and other types of risk.

Table 1. *Risks for heating companies*

Types of risk	**Threads**	**Relationships with other types of risk**
General risks for economy		
interest-rate risk	– volatility of financial markets – return on capital – structure of owned loans	– credit risk – asset risk – liquidity risk
exchange-rate risk	– exchange rate change – change in the economic policy of a country of currency – social unrest in the country's currency – sentiment in the capital market	– risk of changes in the fuel price – liquidity risk – risk of an increase in transport costs – risk of reducing income
force majeure risk	– natural disasters – fuel supply blockage – wars and armed conflicts – strikes	– technological risk – risk of failure – sales volume risk
Industry risks		
risk of changes in fuel prices	– domestic coal mining – demand for fuels on the world market – fuel supply on the domestic and world market	– risk of an increase in operating costs – risk of reducing profit – liquidity risk – risk of reducing expenditure on modernization
risk of changes in prices of heat	– tariff policy of the ERO	– liquidity risk – credit risk – interest rate risk
risk of decreases in demand for district heat	– increasing the share of renewable energy sources – thermo-modernization of buildings – economic situation of recipients – population migration	– sales volume risk – risk of reducing income – liquidity risk
risk of competition	– development of distributed energy – biogas plants and biomethane	– sales volume risk
regulatory and legal risk	– privatization and consolidation of enterprises in the industry – changes in environmental standards – introducing additional requirements	– risk of changes in the price of heat – structural risk – liquidity risk – image loss risk

Table 1 cont.

Types of risk	Threads	Relationships with other types of risk
environmental risk	– exceeding environmental standards – delivery of inferior quality fuels	– risk of paying fines for air pollution – risk of incurring expenses for new purification installations – risk of incurring the costs of purchasing additional emission allowances
	Specific risks	
risk of fuel supplies	– delay in deliveries due to the fault of the seller – reduction in production due to failure – bad weather conditions – delays in deliveries from imports due to the fault of the importer – delays in import deliveries due to other factors	– sales volume risk – risk of civil lawsuits
fuel-quality risk	– fuel calorific value lower than assumed – higher than assumed content of admixtures	– technological risk – environmental risk
technological risk	– faults in the operation of machines and devices – human errors in their operation	– risk of failure – risk of production stoppage – ecological risk
credit risk	– no charges for the delivered heat – delay in charges for heat	– interest rate risk – risk of reducing income – liquidity risk
weather risk	– unusually high temperatures during the heating period – low wind speed – low air humidity	– sales volume risk – risk of reducing income – liquidity risk
internal risk	– inadequate qualifications of employees – bad management system – lack of control mechanisms – deliberate damage to property, plant and equipment – fuel thefts – starting a fire – disruptions to process control systems – hacker attacks on the IT system	– financial risk – risk of production stoppage – ecological risk – image loss risk – risk of reducing income

6. RISK ANALYSIS FOR HEATING COMPANIES AND DISCUSSION

After identifying the risks, they should be analyzed by taking the importance of each into account. Probability and effect are the basic measures in risk analysis; the first can be defined as the possibility of the occurrence of a given threat, and the second as the consequences of its occurrence. Hopkin (2017, p. 22) developed the basic risk analysis with the following parameters:

- time of impact (short- or long-term),
- degree of control entity has over risk (low or high),
- assignment to appropriate risk class (general economic, industry, and specific).

Table 2 presents a qualitative risk analysis for heating companies in Poland. The following designations have been adopted:

- in terms of duration of effect – short-term (+), medium-term (++), long-term (+++),
- in terms of probability, degree of exposure, and degree of control – low (+), medium (++), high (+++) degree of exposure or control.

Table 2. *Qualitative risk analysis for heating companies*

Type of risk	**Duration of effect**	**Probability**	**Degree of exposure**	**Degree of control**
General risks for economy				
interest-rate risk	++	+	+ + +	+
exchange-rate risk	+ + +	++	+ + +	+
force majeure risk	++	+	+	–
Industry risks				
risk of changes in fuel prices	+ + +	+ + +	+ + +	+
risk of changes in prices of heat	+ + +	+	++	+
risk of decreases in demand for district heat	++	++	+	–
competitive risk	++	++	++	–
regulatory and legal risk	+ + +	++	+	–
environmental risk	++	+	+	++
Specific risks				
risk of fuel supplies	+	+	++	++
fuel-quality risk	++	++	+	+
technological risk	++	+	+ + +	++
credit risk	+	+	+	+ + +
weather risk	+	+ + +	++	+
internal risk	++	+ + +	+ + +	+ + +

According to the criterion of the duration of the respective impacts, the following risks are most dangerous: exchange-rate risk, the risk of changes in fuel and heat prices, and regulatory and legal risk. Taking the probability of the risk into account, the highest rating was the possibility of risk of fuel price changes, weather risk, and the group of internal risks. It was also found that heating companies are most exposed to exchange-rate risk, interest-rate risk, risk of changes in fuel prices, technological disturbances, and internal risks. The last parameter that was evaluated was the possibility of risk control. In this sense, the most difficult to deal with are the following risks: force majeure, risk of decreases in demand for district heat, competition risk, and regulatory and legal risk. It should be emphasized that a company has no influence on the general economic risk, the typical risks for the industry are difficult to control, and specific risks are the most conrollable kind of risks (provided that the company's managers are aware of them). This does not mean that they should not worry about other risks. All types of risk should be covered by a coherent management system and integrated with an enterprise management strategy that is aimed at achieving set goals.

So what actions can be taken in relation to the individual risks? Several solutions will be provided below.

To avoid credit risk, the amounts of trade credits should be minimized, and one's credit portfolio should be diversified. There are several solutions for interest-rate risk: not taking out loans unless necessary, applying for fixed interest rates on loans (this enables long-term planning), and using derivatives. In the case of exchange-rate risk, it is recommended to control the structure of foreign currency assets and liabilities, conclude short-term contracts, and use derivatives. In turn, to reduce the exposure to the risk of changes in fuel prices, it is recommended to diversify the the directions of supplies, conclude contracts for short periods, introduce fuel diversification, and maintain emergency reserves of fuels.

7. SUMMARY AND CONCLUSIONS

World population growth, technological development, and urbanization all contribute to increases in the demand for raw materials and energy (electricity and heat). District heating is most-developed in those countries with marked changes in their seasons (including Poland); this determines the energy security of the country during the period from fall to spring. For this reason, the stability of the entities in the heating industry has attracted interest among scientists and practitioners. However, this stability can be disrupted by external and internal threats.

This is why it is so important to develop risk-management strategies in heating companies. The implementation of the strategy will depend on the organizational culture, management style, risk awareness at all levels of management and in all organizational units, speed of the information exchange among them, and information quality. An important role is played by the efficient monitoring and control of the implementation of activities under a strategy (or its newer version) as adapted to new market conditions. This means that a developed strategy may need frequent modifications (even though it covers a long-term perspective). The monitoring team decides when and how the strategy should be modified.

Risk management should support the process of creating value for shareholders and implementing a company's business strategy. This is possible because of the maintenance of the level of risk at a level that is acceptable to the company's stakeholders and the limitation of the impacts of the risk factors on the company's cash flow and financial results. All actions under a risk-management strategy must be proactive; that is, actions that are taken in advance in any event of an incident. They must also be comprehensive; i.e., focused not only on reducing the effects of threats but also on taking advantage of opportunities. The strategy will depend on the general attitude of the managers, who may choose whether to seize opportunities or to reduce threats. Importantly, for a risk-management strategy to be effective, it must be built on the basis of scientific principles and the recommendations of organizations that specialize in this topic.

The qualitative analysis that was carried out here will be the basis for creating strategies for managing various types of risk, using both traditional methods and the most modern ones (e.g., derivatives that are exposed to commodity prices, interest rates, exchange rates, credit risk, or air temperatures).

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Heuristic Algorithm for Lot Sizing and Scheduling on Identical Parallel Machines

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Abstract. This paper presents a new heuristic algorithm for the task of lot sizing and scheduling for identical parallel machines. The new algorithm is based on the rolling-horizon approach and the fix-and-relax decomposition technique. Two variants of the algorithm are finally proposed for solving the problem of lot scheduling with parallel machines where the number of products and machines is greater than that of the machines. A computational experiment has been conducted for a group of 30 data sets. The results showed that the new algorithm efficiently provided good solutions for tasks with large numbers of machines and products.

Keywords: lot sizing, lot scheduling, identical parallel machines, heuristics, algorithm

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JEL Classification: O14, D24

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1. INTRODUCTION

Scheduling the deliveries of raw materials, products, and semi-finished products as well as the receipts of finished products is crucial in production company management. Setting the dates for supply and distribution depends on a production plan that determines the dates and demands for particular raw materials and semi-finished products as well as the deadlines for the completions of particular production stages. This allows us to indicate the deadlines for the productions of semi-finished products and finished goods to be delivered to customers. Therefore, logistic flow management is related to the lot sizing and scheduling that enable us to define the volumes of production lots and plan production slots and changeovers.

Lot sizing and scheduling is a problem in the field of production engineering, which deals with the development of scheduling methods for different production systems (among other things). Over the last few decades, many methods of solving

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lot sizing and scheduling problems have been developed (Jans & Degraeve, 2008; Pochet & Wolsey, 2006).

This paper presents a new heuristic algorithm for the task of *lot sizing and scheduling* for identical parallel machines.

In such a task, a product lot is manufactured in order to meet a deterministic demand within a finite time frame that is divided into periods. The production runs on several identical parallel machines with finite production capacities. The production output of a given machine must not exceed its capacity throughout the entire period. If a given machine manufactures various products, a changeover is required between the lots of the different types. Each changeover generates costs and consumes time.

The aim of production scheduling is to develop a production plan that minimizes inventory and production costs. Compromise solutions that minimize total costs are desired. On one hand, large lots reduce the number of changeovers, but on the other hand, they require high inventory levels and result in increased costs. With smaller production lots and lower inventory levels, the costs are moderate; however, a large number of lots means that the costs of the changeovers are frequently incurred.

The aim of this study was to develop a heuristic algorithm for identical parallel machines. Haase (1994) presented a randomized algorithm that makes backward decisions about changeovers and lot sizing (backward add-method). Due to the random nature of the decision-making rules, this algorithm provides many solutions; the best ones have been selected. This paper shows that Haase's algorithm can be applied in tasks with many identical parallel machines.

2. MODELS OF TASKS WITH PARALLEL MACHINES

In a task of lot scheduling for parallel machines, the production runs on a group of identical parallel machines with finite capacity. The products are manufactured to meet a deterministic demand on a certain scheduling horizon divided by a given number of periods. The production output of a given machine must not exceed its capacity during any scheduled period. When switching production from one product onto another, a machine changeover is required. Each changeover generates costs and takes time; its time must be shorter than the duration of the whole period. The costs and times of changeovers do not depend on the orders in which the lots are produced.

The aim of production scheduling is to develop a production plan that minimizes inventory costs and production costs.

There are two ways to formulate the mixed-integer programming (MIP) models for the tasks of lot sizing and scheduling on identical parallel machines. In the first approach, the model defines the binary variables separately for each machine (Kimms & Drexl, 1998). The binary variables indicate whether a given machine meets certain conditions that are required in the task; e.g., the machine is ready to produce a given product in a given period of time or it is not.

Such a formulation of a task results in a situation where we get many practically identical symmetrical solutions that differ only in terms of the machine numbers. Analyses of symmetrical solutions only increase the calculation workload in the branch-and-bound method, as the same solution is evaluated multiple times.

In the second approach, the task formulation is based on the aggregation of machines. The binary variables are replaced by integer variables that indicate the number of machines that fulfill certain requirements in a task; e.g., the number of machines that have been changed over and are ready to produce a given lot in a specific period.

Binary variables y_{ijt} and z_{jt} can be replaced with aggregated variables where $y_{jt} = \sum_{i \in M} y_{ijt}$ and $z_{jt} = \sum_{i \in M} z_{ijt}$. Lasdon (1971) used such variables and formulated a DLSP model with identical parallel machines. Table 1 contains a list of the basic symbols that are used to describe the models.

Table 1. *List of basic symbols*

Data	
$T = \{1, ..., T\}$ – set of periods, T – number of periods,	
$N = \{1, ..., N\}$ – set of products, N – number of products,	
$M = \{1, ..., \mu\}$ – set of machines, μ – number of machines.	
Parameters	
d_{jt}	– demand for product j in period t,
C_t	– length of period t,
p_j	– unit time of manufacturing product j,
S_j^T	– time of changeover to product j,
I_{j0}	– initial inventory of product j,
S_j^C	– cost of changeover to product j,
h_j	– unit cost of product j inventory.
Continuous variables	
a_{jkt}	– relative part of total capacity of all machines whose productions of product j are stopped and productions of product k are started in period t, reserved for product k after changeover,
b_{jkt}	– relative part of total capacity of all machines whose productions of product j are stopped and productions of product k are started in period t, reserved for product j before changeover,
I_{jt}	– inventory of product j by end of period t,
$q_{(i)jt}$	– production lot of product j in period t (on machine i).
Binary and integer variables	
f_{jkt}	– number of machines changed over from product j to product k in period t,
f_{jjt}	– number of machines producing product j in period $t-1$ that are ready to produce product j in period t.
$y_{jt} = 1$ – machine ready to produce product j in period t, 0 – otherwise,	
$z_{jt} = 1$ – production of product j is started on machine in period t, 0 – otherwise.	
$y_{ijt} = 1$ – machine i is ready to produce product j in period t, 0 – otherwise (y_{ij0} – initial state of machine),	
$z_{ijt} = 1$ – production of product j is started on machine i in period t, 0 – otherwise.	

The PLSP binary model cannot be directly transformed into the integer model with parallel machines; transforming the PLSP task in this way would be incorrect.

Kaczmarczyk (2011) presented an appropriate model called PLSP/F. The variables of flow f_{jkt} were the only integers in this model. These indicated the flow of the machine availability "units" between two products; this means that a certain number of machines that were prepared to manufacture a particular product in a previous period has been changed over and is now ready to manufacture another product. The variables of flow indicate the order of the manufacturing of particular products in a given period. The model can be formulated as follows:

$$\min \sum_{t\in T}\sum_{j\in N}\Big(h_{jt}I_{jt} + \sum_{k\in N:j\neq k} S^C_{jk} f_{jkt}\Big) \tag{1a}$$

$$I_{jt-1} + q_{jt} = d_{jt} + I_{jt}, \qquad t\in T, j\in N, \tag{1b}$$

$$\frac{p_j}{C_t q_{jt}} \le f_{jjt} + \sum_{k\in N:j\neq k}(b_{kjt} + a_{jkt}), \quad t\in T, j\in N, \tag{1c}$$

$$b_{jkt} + a_{jkt} = f_{jkt}\Big(1 - \frac{S^T_{jk}}{C_t}\Big), \qquad t\in T, (j,k)\in N^2 : j\neq k, \tag{1d}$$

$$f_{jk0} = 0, \qquad t\in T, j\in N, j\neq k, \tag{1e}$$

$$f_{jj0} = y_{j0}, \qquad t\in N, \tag{1f}$$

$$\sum_{k\in N} f_{kjt-1} = \sum_{k\in N} f_{jkt}, \qquad t\in T, j\in N, \tag{1g}$$

$$\sum_{(j,k)\in N^2} f_{jk0} = m, \tag{1h}$$

$$q_{ijt}, I_{jt} \ge 0, \qquad t\in T, j\in N, \tag{1i}$$

$$a_{jkt}, b_{jkt} \in [0,m], \qquad t\in T, (j,k)\in N^2 : j\neq k, \tag{1j}$$

$$f_{jkt} \in \{0,\dots,m\}, \qquad t\in T, (j,k)\in N^2. \tag{1k}$$

Limitations (1b)–(1d) ensure the correct values of the variable production and inventory volumes. Limitations (1e)–(1h) are accountable for the correct integer values of the flow variables.

In the case of a large number of products, this model is difficult to solve using standard MIP methods; however, the task becomes easier with an increased number of identical machines.

In tasks with binary variables, an increased number of machines results in the larger scope of a task. So far, no effective heuristic algorithm has been developed that solves this problem (Kaczmarczyk, 2011).

LSPIPM (lot sizing and scheduling problem with identical parallel machines), the model that was proposed by Beraldi et al. (2008), can only be used to effectively solve small tasks; therefore, a heuristic algorithm using a rolling horizon approach and fix-and-relax decomposition technique was formulated.

Mehdizadeh et al. (2015) presented a model of linear integer programming for the problem of lot sizing and scheduling on parallel machines provided that changeover times do not depend on the sequence of the lots that are allocated to the machine.

In this task, Mehdizadeh proposed a metaheuristic approach based on the vibration-damping optimization (VDO) algorithm.

To solve the problem of lot sizing and scheduling with identical parallel machines with fixed dates of the deliveries of finished goods, Mensendiek et al. (2015) formulated a mathematical model using the model for the task of allocation with binary variables; the model was aimed at minimizing the total delay. Due to the NP-hardness of the task, a Tabu-search algorithm and a hybrid genetic algorithm were developed to solve larger tasks.

The presented examples of models for the lot sizing and scheduling problem for one machine and parallel machines show that the formulated mathematical models are usually based on allocation tasks. This type of task is characterized by a large number of variables, and binary variables are predominant in the models; therefore, the mathematical models allow for optimal solutions in small tasks due to the NP-hardness.

The real problems regarding production scheduling are most often large tasks; solutions to these problems are expected to be found quickly. Therefore, appropriate heuristic and metaheuristic algorithms are being developed in order to solve these problems effectively – especially genetic and Tabu-search algorithms.

3. NEW HEURISTICS FOR TASKS WITH PARALLEL MACHINES

This algorithm is an adaptation of the BAPLSP algorithm that was developed by K. Haase for the problem of lot sizing and scheduling for one machine. There are two variants of the algorithm: one for a task where the number of products is greater than the number of machines, and another for a task where the number of products is smaller or equal to the number of available machines.

Similarly to the BAPLSP algorithm for a single machine task, let us assume (like Haase, 1994) that the production capacity C_t in all scheduling periods t is equal to 1 and the production times of particular products p_j are also equal to 1.

The solution is calculated backwards – from the last period of lot scheduling T to the first period. We always plan the maximum possible production volume of a given product, in a given period, and on a particular machine. The obtained solution may be unacceptable if some part of the demand for a certain product remains unscheduled.

In each period of solution search $\tau \in T$, the products with unsatisfied demands are allocated to consecutive machines. The product is allocated to a machine that offers spare capacity before or after the changeover.

The products are drawn from a set of currently available N'_τ products. The set is continuously updated for each machine and a given period τ. Each product in this set has a certain priority.

For a drawn product j and a given machine m in period τ, the production lot is scheduled backwards. The initial period of lot scheduling t is equal to period τ. The lot is scheduled backwards in a specific time slot $[t';\tau]$, where period t' indicates a number of lot-scheduling periods that is equal to $(\tau - t')$. The method of calculating the number of lot-scheduling periods is thoroughly described in the section of this paper that is dedicated to the new algorithm.

General description of algorithm's operation

Draw value of control parameters
$\tau = T$
As long as some part of demand has not been scheduled and $\tau > 0$
    For each machine m
        Draw product j, where $j \in N'_\tau$
        Sequentially for $t = \tau;\ t = \tau - 1; \dots;\ t = t'$
            Schedule lots for product j
    $\tau = \tau - 1$
Show solution

The value of total unsatisfied demand TD (Haase, 1994) at the beginning of the algorithm's operation is equal to the sum of the total demand for the products. During the algorithm's operation, the value of total unsatisfied demand TD is reduced by the number of produced pieces $\widetilde{q}_{jt}$ after deciding on the quantity of produced pieces $\widetilde{q}_{jt}$ of a given product j in a given period t. If the value of the total unsatisfied TD demand is greater than zero ($TD \geq 0$) after the algorithm's operation is finished, this means that the given quantity of products will not be produced on time; hence, the received solution is unacceptable.

The value of the total unsatisfied demand for a given product j is equal to the value of unsatisfied demand $\widetilde{D}_{j0}$ (Haase, 1994) in period 0; i.e., the total number of products that should be planned for production in Periods 1 through T.

Example 1. Calculate the value of unsatisfied demand $\widetilde{D}_{j,t}$ and the value of the total unsatisfied demand TD for four scheduling periods $t = (27, \dots, 30)$ for a task with two products (k and l).

t	...	27	28	29	30
d_{kt}	...	1	2	3	4
q_{kt}	...	-	4	-	-
d_{lt}	...	5	6	7	8
q_{lt}	...	-	10	-	-

Calculated values:

$\widetilde{D}_{k,30} = (4 - 0) = 4$
$\widetilde{D}_{k,29} = \widetilde{D}_{k,30} + (3 - 0) = 7$
$\widetilde{D}_{k,28} = \widetilde{D}_{k,29} + (2 - 4) = 5$
$\widetilde{D}_{k,27} = \widetilde{D}_{k,28} + (1 - 0) = 6$
$\widetilde{D}_{l,30} = (8 - 0) = 8$
$\widetilde{D}_{l,29} = \widetilde{D}_{l,30} + (7 - 0) = 15$
$\widetilde{D}_{l,28} = \widetilde{D}_{l,29} + (6 - 10) = 11$
$\widetilde{D}_{l,27} = \widetilde{D}_{l,28} + (5 - 0) = 16$
$TD = 6 + 16 = 22$

■

Let us assume that $\widehat{d}_{jt}$ is the unsatisfied demand for a given product j in a given period t. At the beginning of the algorithm's operation, its value is equal to demand d_{jt}. If a decision is made to produce a lot of product j in quantity $\widetilde{q}$ in period t, unsatisfied demand $\widehat{d}_{jt}$ is reduced by $\min(\widehat{d}_{jt}, \widetilde{q})$.

Thus, if a lot size $\widetilde{q}$ is greater than unsatisfied demand $\widehat{d}_{jt}$, the products that are manufactured in excess of the needs in period t will cover the demands of the subsequent periods. Due to the increasing warehousing costs, it was assumed that those products that were produced in period t would cover the demands in consecutive periods as much as possible. The procedure is continued until all of the surplus production from period t is used.

Therefore, the values of the unsatisfied demands to be planned for future periods $\widehat{d}_{jt+1}, \widehat{d}_{jt+2}, \ldots, \widehat{d}_{jT}$ also change. If lot size $\widetilde{q}$ is greater than unsatisfied demand $\widehat{d}_{jt}$ in the considered case, the unsatisfied demand $\widehat{d}_{jt}$ in period t will certainly be equal to 0, while it will decrease correspondingly to the volume of the planned lot $\widetilde{q}$ in periods $t+1$ and $t+2, t+3 \ldots$.

Lot size $\widetilde{q}$ deliberately does not have any subscript with a product or period because, during the algorithm's operation, we might decide to create a production lot of a given product more than once in a given period (in this situation, the planned production $q_{jt} = \widetilde{q}' + \widetilde{q}'' + \ldots$ for product j in period t).

The unsatisfied demand $\widehat{d}_{jt}$ to be scheduled in a time slot $n = [t, \ldots, T]$ and the decision to produce a certain lot size $\widetilde{q}$ of a given product j in period t can be described as a recurrence relationship:

$$\begin{cases} \widehat{d}_{jn} \leftarrow \max(0, \widehat{d}_{jn} - \widetilde{q}); \qquad \widehat{q}_n \leftarrow \max(0, \widetilde{q} - \widehat{d}_{jn}); \\ \\ \widehat{d}_{j,n+1} \leftarrow \max(0, \widehat{d}_{j,n+1} - \widehat{q}_n); \; \widehat{q}_{n+1} \leftarrow \max(0, \widehat{q}_n - \widehat{d}_{j,n+1}); \end{cases} \tag{2}$$

Example 2. Calculate the values of unsatisfied demand $\widehat{d}_{k,2}$ for a certain product k and $\widetilde{q} = 9$ in the 26^{th} period of production. The table presents the initial values of unsatisfied demand $\widehat{d}_{kt}$ to be scheduled in the particular periods.

t	...	25	26	27	28	29	30
$\widehat{d}_{kt}$	...	5	5	3	2	1	2

$n = 26$

$\widehat{d}_{k,26} \leftarrow \max(0; 5-9)$ $\qquad$ $\widehat{q}_{26} \leftarrow \max(0; 9-5)$

$\widehat{d}_{k,26} = 0$ $\qquad$ $\widehat{q}_{26} = 4$

$n = 27$

$\widehat{d}_{k,27} \leftarrow \max(0; 3-4)$ $\qquad$ $\widehat{q}_{27} \leftarrow \max(0; 4-3)$

$\widehat{d}_{k,27} = 0$ $\qquad$ $\widehat{q}_{27} = 1$

$n = 28$

$\widehat{d}_{k,28} \leftarrow \max(0; 2-1)$ $\qquad$ $\widehat{q}_{28} \leftarrow \max(0; 1-2)$

$\widehat{d}_{k,27} = 1$ $\qquad$ $\widehat{q}_{27} = 0$

$n = 29$

$\widehat{d}_{k,29} \leftarrow \max(0; 1 - 0)$ $\quad\quad\widehat{q}_{28} \leftarrow \max(0; 0 - 1)$

$\widehat{d}_{k,29} = 1$ $\quad\quad\widehat{q}_{29} = 0$

$n = 30$

$\widehat{d}_{k,30} \leftarrow \max(0; 2 - 0)$ $\quad\quad\widehat{q}_{28} \leftarrow \max(0; 0 - 2)$

$\widehat{d}_{k,30} = 2$ $\quad\quad\widehat{q}_{30} = 0$

■

For each period of backward scheduling τ, let $\widehat{D}_{jt}$ indicate the unsatisfied demand for product j in any given period t summed up in a time slot $W = [\tau - \lambda_j, \ldots, T]$:

$$\widehat{D}_{j,t} = \begin{cases} \sum_{s=t}^{T} \widehat{d}_{js}, & \text{if} \quad t \in W, \\ \widehat{d}_{jt}, & \text{else}, \end{cases} \quad t = 1, \ldots, T. \tag{3}$$

A random parameter λ_j for a given product j determines the maximum duration of the backward lot scheduling that results in a build-up of inventory. Therefore, when planning lots for a certain backward scheduling period τ based on unsatisfied demand $\widehat{D}_{j,t}$, the costs of warehousing the produced goods will occur only between periods $\tau - \lambda_j$ and τ.

Example 3. Calculate the value of the unsatisfied demand $\widehat{D}_{k,t}$ for a given product k in the 29$^{\text{th}}$ period of the solution search and with the value of a random parameter $\lambda_k = 2$. The table presents the initial values of unsatisfied demand $\widehat{d}_{kt}$ to be scheduled in the particular periods.

t	...	25	26	27	28	29	30
$\widehat{d}_{kt}$	...	5	5	3	2	1	2

$W = [29 - 2, \ldots, 30] = [27, \ldots, 30]$

...

$\widehat{D}_{k,25} = 5$ $\quad\quad(t = 25) \notin W$

$\widehat{D}_{k,26} = 5$ $\quad\quad(t = 26) \notin W$

$\widehat{D}_{k,27} = 3 + 2 + 1 + 2 = 8$ $\quad\quad(t = 27) \in W$

$\widehat{D}_{k,28} = 2 + 1 + 2 = 5$ $\quad\quad(t = 28) \in W$

$\widehat{D}_{k,29} = 1 + 2 = 3$ $\quad\quad(t = 29) \in W$

$\widehat{D}_{k,30} = 2$ $\quad\quad(t = 30) \in W$

■

For each period of backward scheduling τ, let $\breve{R}_{j\tau}$ indicate a certain expected reserve that is saved for a given product j in time slot $W = [\tau - \psi, \ldots, T]$:

$$\breve{R}_{j\tau} = \sum_{s \in W} \widehat{d}_{js}, \quad t \in W. \tag{4}$$

If a random parameter $\psi = 0$, then an expected reserve $\breve{R}_{j\tau}$ for a given product j in scheduling period τ equals total unsatisfied demand $\breve{R}_{jt} = \sum_{s\in[t,\ldots,T]} \widehat{d}_{js}$.

Example 4. Calculate the values of the expected saved inventory $\breve{R}_{k29}$ of a given product k in the 29^{th} scheduling period for random values of parameter $\psi = (0,2)$. The table presents the initial values of unsatisfied demand $\widehat{d}_{kt}$ in the particular periods.

t	...	25	26	27	28	29	30
$\widehat{d}_{kt}$	...	5	5	3	2	1	2

$\psi = 0$

$\breve{R}_{k,29} = 1 + 2 = 3$ $\quad W = [29 - 0, 30] = [29, 30]$

$\psi = 2$

$\breve{R}_{k,29} = 3 + 2 + 1 + 2 = 8$ $\quad W = [29 - 2, 30] = [27, \ldots, 30]$

■

Let $r_{j\tau}(i)$ indicate the value of the lost profit if product j is not selected (assuming that a given machine is available to produce product i in period in period τ and the random parameter is σ):

$$r_{j\tau}(i) = \begin{cases} (1-\gamma)\breve{R}_{j\tau}h - \gamma S_j^C & \text{for } j \neq i; \widehat{D}_{j,\tau} \geq \sigma - 1 \\ (1-\gamma)\breve{R}_{j\tau}h & \text{for } j = i; \widehat{D}_{j,\tau} \geq \sigma - 1 \\ -\infty & \text{otherwise} \end{cases} \tag{5}$$

The lost profit value is defined in the same way as in Haase's (1994) work on the problem of lot sizing and scheduling for production on a single machine.

For each machine m and product j in period τ, the value of the lost profit is calculated as the sum of the saved (positive) costs of inventory $\breve{R}_{j\tau}h_j$ and the incurred (negative) costs of launching a new production lot S_j^C.

It is possible to calculate the value of the lost profit and allow for the possibility of drawing a product to be scheduled for production if the unsatisfied demand $\widehat{D}_{j\tau}$ for a given product j in a given period τ is higher than minimum-required unsatisfied demand $\sigma - 1$.

Random parameter σ indicates the minimum required unsatisfied demand $\widehat{D}_{j,\tau}$ for a given product j in a given period τ. A parameter that is equal to 1 does not limit the possibility of product selection. A parameter that is equal to 2 means that the demand for a given product is sufficient to schedule its production for at least one period. Respectively, a parameter that is equal to 3 – not less than 2 periods, etc.

Parameter γ determines the weights of the incurred costs. Depending on its value, this parameter allows us to control the lot size (parameter γ has a value within a range of $[0; 1]$). If the value of parameter γ is close to 1, relatively large production lots should be expected. As the changeover cost has a significant influence on the value of lost profit $r_{j\tau}$, those products with changeover costs that are lower than S_j^C are more likely to be selected.

However, if parameter γ is close to 0, we can expect a greater number of relatively small production lots to be created. The savings on inventory costs $\breve{R}_{j\tau}h_j$ are considered to be more important.

Let $\rho_{j\tau}(i)$ be the criterion for selecting a product to be manufactured. Similarly to the case with one machine that was presented by Haase (1994):

$$\rho_{j\tau}(i) = \begin{cases} 0; & \text{for } \; r_{j\tau} = -\infty, \\ (r_{j\tau} - \min\{r_{k\tau} | k \in N \wedge r_{k\tau} > -\infty\} + \varepsilon)^{\delta}; & \text{else,} \end{cases} \tag{6}$$

where $\delta \geq 0, \varepsilon > 0$.

Those products that are admitted to the draw have values of decision criterion $\rho_{j\tau}$ that are greater than 0 and are "compared" with the worse options (those products with lower $\rho_{j\tau}$ values). If the value of decision criterion $\rho_{jt}(i)$ is equal to 0, the product is not taken into account in the draw. A high value of the decision criterion means that the product is more likely to be selected. Therefore, the value of parameter $\delta > 1$ increases the chances of products with higher lost profit values $r_{j\tau}(i)$; conversely, the chances decrease when $\delta < 1$. Furthermore, parameter $\epsilon > 0$ makes it probable for a product with the worst lost profit value $\min\{r_{k\tau}(i) | k \in N \wedge r_{k\tau}(i) > -\infty\}$ to be drawn.

The products are drawn based on the calculated $\rho_{j\tau}(i)$ values.

Example 5. Calculate the criterion values for three products (j, k, l) in the 27^{th} period of a solution search and a given machine m that is available to produce product k in this period. The inventory costs and changeover costs in this period are equal for all of the products and are 2 and 10, respectively.

t	...	25	26	27	28	29	30
$\widehat{d}_{jt}$	...	0	0	0	0	0	1
$\widehat{d}_{kt}$	...	5	5	3	2	1	2
$\widehat{d}_{lt}$	...	2	7	0	0	0	0

Parameter	Value
ψ	2
γ	0.3
δ	2
ϵ	1
σ	2

$\widehat{D}_{j,27} = \widetilde{D}_{j,27} = 0 + 0 + 0 + 1 = 1$
$\widehat{D}_{k,27} = \widetilde{D}_{k,27} = 3 + 2 + 1 + 2 = 8$
$\widehat{D}_{l,27} = \widetilde{D}_{l,27} = 0 + 0 + 0 + 0 = 0$

$\breve{R}_{j,27} = 0 + 0 + \widehat{D}_{j,27} = 1$
$\breve{R}_{k,27} = 5 + 5 + \widehat{D}_{k,27} = 18$
$\breve{R}_{j,27} = 2 + 7 + \widehat{D}_{l,27} = 9$

$r_{j,27}(k) = 0.7 \cdot 1 \cdot 2 - 0.3 \cdot 10 = -1.6$
$r_{k,27}(k) = 0.7 \cdot 18 \cdot 2 = 25.2$
$r_{l,27}(k) = -\infty$

$\rho_{j,27}(k) = (-1.6 + 1.6 + 1)^2 = 1$
$\rho_{k,27}(k) = (25.2 + 1.6 + 1)^2 = 772.84$
$\rho_{l,27}(k) = 0$

■

The BAPLSP/F$^{N>\mu}$ algorithm for lot sizing and scheduling for parallel machines has been developed for those cases where the numbers of products are greater than the numbers of machines.

BAPLSP/F$^{N\leq\mu}$ is an algorithm for those cases where the numbers of products are smaller or equal to the numbers of machines.

The formal description of the stochastic heuristic $BAPLSP/F$ algorithm is as follows for the cases of BAPLSP/F$^{N>\mu}$ and BAPLSP/F$^{N\leq\mu}$:

BAPLSP/F$^{N>\mu}$, $\boxed{\text{BAPLSP/F}^{N\leq\mu}}$:

Initialization of variables:

$\gamma,\ \delta,\ \epsilon,\ \psi,\ \omega,\ \alpha$ {parameter drawing}
$\widetilde{y}_{m\tau} := \varnothing,\ c_{m\tau} := 1,\ m \in M, \tau \in T$ {[no availability to manufacture products, initial production capacity}
$i_m := \varnothing,\ t_m := T,\ m \in M$ {lack of product, we are considering period for machine m}
$q_{j\tau} := 0,\ \widehat{D}_{j\tau},\ TD, \widehat{d}_{j\tau},\ j \in N, \tau \in T$ {initialization of variables}

Algorithm's operation:

$\tau := T$ {starting from last period}
`while` $TD > 0 \wedge \tau > 0$: {start of Loop 1}
  `for` $m \in M$ `if` $\tau \leq t_m$: {one by one for each available machine in period}
    $t_m := \tau$
    `if` $\widetilde{y}_{m,t_m} = \varnothing \wedge t_m < T$
      $\widetilde{y}_{m,t_m} := i_m$ {transfer of machine availability}
    `random` $\sigma_j;\ j \in N$
    `if random uniform` $(0,1) \leq \alpha$:
      $\lambda_j :=$`random`$(\sigma_j, max(\sigma_j + 1, t_m);\ j \in N$
    `else:` $\lambda_j := t_m$
    `determine` $r_{j,t_m}(i_m);\ \rho_{j,t_m}(i_m);\ j \in N$
    `if` $\sum_{j\in N} \rho_{j,t_m}(i_m) = 0$:
      $t_m := t_m - 1$; `continue for next` m
    $\widehat{i}$:=`random proportional to` $\rho_{j,t_m}(i_m)$
    `determine` $\widehat{D}_{\widehat{i},t_m}$
    `while` $\widehat{D}_{\widehat{i},t_m} > 0 \wedge t_m > 0$: {start of Loop 2, schedule for selected product}
      $\widetilde{q} := 0,\ \widetilde{c} := 0$ {initial production volume, demand for production capacity}
      `if` $\widetilde{y}_{m,t_m} = \varnothing \vee \widetilde{y}_{m,t_m} = \widehat{i}$: {cases for changeovers}
        $\widetilde{y}_{m,t_m} := \widehat{i}$
        $\widetilde{q} := \widetilde{c} := \min(c_{m,t_m}, \widehat{D}_{\widehat{i},t_m})$
      `elseIf` $\widetilde{y}_{m,t_m} \neq \widehat{i} \wedge c_{m,t_m} \geq S^T_{i_m}$:
        $\widetilde{y}_{m,(t_m-1)} := \widehat{i}$
        $\widetilde{q} := max(0, min(c_{t_m,m} - S^T_{i_m}, \widehat{D}_{i_m,t_m})$
        $\widetilde{c} := \widetilde{q} + S^T_{i_m}$
      `elseIf` $\widetilde{y}_{m,t_m} \neq \widehat{i} \wedge c_{mt_m} < S^T_{i_m}$:
        $\widetilde{y}_{m,t_m-1} := \widehat{i},\ t_m := t_m - 1$; `continue while`
      $q_{\widehat{i},t_m} := q_{\widehat{i},t_m} + \widetilde{q},\ c_{m,t_m} := c_{m,t_m} - \widetilde{c}$
      `determine` $\widehat{d}_{\widehat{i},t_m},\ \widehat{D}_{\widehat{i},t_m},\ TD$

33 **if** $c_{m,t_m} \le \omega \; \boxed{\vee\, t_m \ge \tau - \lambda_{\widehat{i}}^{\;*}}$: {Usage of production capacity, *–extension for BAPLSP/F$^{N>\mu}$ }

34 $t_m := t_m - 1$; **else: break**

35 $i_m := \widehat{i}$

36 **if all** $t_m < \tau$: $\{\tau := \tau - 1$ **and** $\widehat{D}_{j\tau} = \widehat{D}_{j,(\tau+1)} + \widehat{d}_{j\tau};\ j \in N\}$

In Line 1, all of the values of the fixed parameters are drawn for the entire run of the solution-search algorithm.

In Line 2, a given machine m is not ready ($\widetilde{y}_{mt}$) to produce any product in a given period t at the beginning of the algorithm's operation. The remaining production capacity c_{mt} of a given machine m in a given period t is equal to the total available production capacity in period t, as we are not producing anything yet.

In Line 3, the last product i_m that is produced on machine m is undefined. The period of machine availability t_m in which we can start planning products is equal to the last planning period T.

In Line 4, the initial production volume of a given product j in a given scheduling period τ is equal to zero. The volume of the unsatisfied demand $\widetilde{D}_{j,t}$ for particular products and periods is calculated, as is total unsatisfied demand TD. The unsatisfied demand $\widehat{D}_{jt}$ for particular products and periods is calculated for time slot $W = [\tau, \dots, T]$. The volume of the unscheduled $\widehat{d}_{j\tau}$ products is initially equal to demand d_{jt}.

In Line 6, the scheduling starts from the last period $t = T$. The solution search continues until the total TD demand is met.

In Line 7, the production lots of given products are successively launched on all machines m. The schedule for a given machine m is created (provided that it is available in the given period τ – it is not producing another product), and the available production capacity allows us to plan the production in Periods 1 through τ.

In Lines 9 and 10, if a given machine m does not produce any product for a certain number of periods subsequent to τ and the machine is to produce a certain product in a period after the non-production period, then the machine is available ($\widetilde{y}_{m,t_m}$) to produce the last product i_m in period τ.

In Lines 11–14, two parameters are drawn (σ_j, and λ_j) for all products $j \in N$.

If the unsatisfied demand for product j in period τ is greater than or equal to σ_j, a new production lot will be created (the lot will not be shorter than σ_j) – compare this to 5.

The λ_j parameter determines the length of the backward scheduling of a given product that results in inventory build-up. The value of parameter λ_j is drawn with a certain probability of α; it is a random input parameter for a given run of the new algorithm. This determines the probability of a situation where we draw the value of parameter λ_j; otherwise, parameter λ_j is equal to the maximum possible length of the backward scheduling of a given product. Random values of parameter λ_j for product j belong to a range that is limited by the minimum length of created production lot σ_j and the maximum possible length of the backward scheduling of a given product j.

In Line 15, the value of lost profit $r_{jt}(i_m)$ and the value of the criterion for product-selection decision $\rho_{jt}(i_m)$ are calculated for all products $j \in N$ for a given machine m and the availability of this machine for the production of product i_m in period τ ($t_m = \tau$) – compare Formulae (5) and (6).

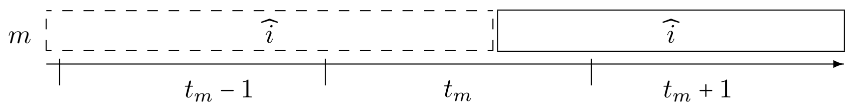
In Lines 16–18, product $\widehat{i}$ is drawn for lot scheduling. If there is no product $\sum_{j \in N} \rho_{jt} = 0$ that is available to be drawn for a given machine m in a given period τ, the solution for the next machine is created.

In Line 19, we calculate the total unsatisfied demand $\widehat{D}_{\widehat{i},t_m}$ for a drawn product $\widehat{i}$ on machine m in a period of lot scheduling for a given machine t_m (at the beginning of the backward lot sizing of product i, this period is equal to period τ).

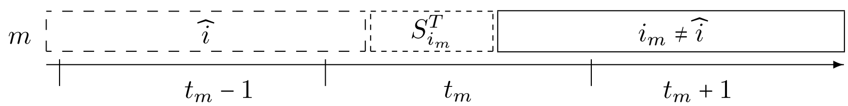
In Line 20, we schedule the production lots backwards until the unsatisfied demand $\widehat{D}_{\widehat{i},t_m} > 0$ for product $\widehat{i}$ in a period of lot scheduling t_m for a given machine m is fulfilled. Lot-sizing period t_m must be within planning horizon $t_m \geq 0$; the first period is the last period in the backward scheduling.

In Line 21, we define the size of production lot $\widetilde{q}$ and production capacity demand $\widetilde{c}$ in period t_m; the values are calculated accordingly:

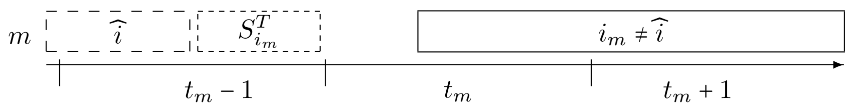
1) When machine m is available to produce the same selected product $\widehat{i}$ (or is not ready to produce any product – this situation occurs only once for each machine; i.e., when the production on a given machine starts for the first time [an illustration of this case is not provided]) (Lines 22–24):



2) When machine m is available to produce a different product than the selected product $\widehat{i}$, and the remaining production capacity c_{m,t_m} of machine m in lot-scheduling period t_m allows for a changeover (Lines 25–28):



3) When machine m is available to produce a different product than the selected product $\widehat{i}$, and the remaining production capacity c_{m,t_m} of machine m in lot-scheduling period t_m does not allow for a changeover (Lines 29–30):



In the first case, the production lot size $\widetilde{q}$ and capacity demand $\widetilde{c}$ in the lot-scheduling period t_m of the selected product $\widehat{i}$ and for a given machine m are calculated on the basis of available production capacity c_{m,t_m} and total demand $\widehat{D}_{\widehat{i},t_m}$.

In this case, we do not use available production capacity c_{m,t_m} to make a changeover. We use all of the capacity to manufacture the product (or to continue the previously started lot of product $\widehat{i}$). The selected machine m is available to produce product $\widehat{i}$ in lot-scheduling period t_m.

In the second and third cases, it is necessary to take the time that is required to make a changeover $S^T_{i_m}$ into account when calculating the value of production capacity demand $\widetilde{c}$ in order to start producing the lots of product i_m later on. In both cases, machine m is available to produce product i_m in period t_m due to a change of products on a given machine m and the resulting changeover $S^T_{i_m}$; in the earlier period $t_m - 1$, it is available to produce a selected product $\widehat{i}$.

In the second case, we still have some spare production capacity $c_{t_m,m} - S^T_{i_m}$ in period t_m after the changeover; we use this to start the production of product $\widehat{i}$ in this period.

In the third case, the available production capacity $c_{t_m,m}$ does not allow for the required changeover S_{i_m} in period t_m; so, the launch of a new lot is only possible in the previous lot-scheduling period $t_m - 1$.

In Line 32, the number of unscheduled products $\widehat{d}_{\widehat{i},t_m}$ and total unsatisfied demand $\widehat{D}_{\widehat{i},t_m}$ for a given product $\widehat{i}$ in lot-scheduling period t_m are calculated anew in case the production of product $\widehat{i}$ in quantity $\widetilde{q}$ is possible in period t_m. The size of unsatisfied TD demand is reduced by the volume of the scheduled production $\widetilde{q}$ of the given product $\widehat{i}$.

In Lines 33 and 34, a decision is made regarding whether or not to continue lot scheduling for product $\widehat{i}$ and given machine m. If the spare production capacity c_{m,t_m} of a given machine m in lot-scheduling period t_m is less than or equal to the assumed permissible level of unused available production capacity ω, the scheduling for a given product $\widehat{i}$ is continued. Otherwise, the scheduling for a given product $\widehat{i}$ in scheduling period t_m is completed.

For those tasks where the numbers of products are less than or equal to the numbers of machines, the production lot is always continued if the lot-scheduling period t_m for machine m fits in a time slot $[\tau - \lambda_{\widehat{i}}, \ldots, \tau]$.

In Line 35, we set the availability of machine i_m (the last product that is produced on the machine) for a given product $\widehat{i}$after completing the (backward) lot scheduling for a given product $\widehat{i}$.

In Line 36, the scheduling returns to the previous period $\tau - 1$ if all of the possible machines $m \in M$ are used in a given period τ. Additionally, the value of total unsatisfied demand $\widehat{D}_{j,\tau-1}$ is updated for each product.

4. COMPUTATIONAL EXPERIMENTS

In order to evaluate the new BAPLSP/F algorithm, computational experiments were conducted for a group of different data sets.

Optimal solutions were sought with GUROBI software (Version 6.0.4); the GUROBI solver is recognized as the state-of-the-art solver for mathematical programming (GUROBI, 2022). The GUROBI solver was designed from scratch using a modern multi-core processor architecture; it allows us to solve tasks that are formulated as LP (linear programming) models, mixed-integer linear programming (MILP) models, quadratic programming (QP) models, etc. The GUROBI solver can solve models with millions of variables and limitations on standard mobile and desktop computers; it also provides interfaces for the majority of the popular programming languages.

The new heuristic algorithm has been implemented in the Python 2.7 programming language. It allows us to code a clear and synthetic program; thus, it is often used in software prototyping. In order to accelerate the process, the calculations were performed with a PyPy compiler, which is a just-in-time compiler (JIT). The first time a function code fragment is called, it compiles it into a machine code (PyPy, 2022). The machine code consists of a sequence of binary numbers that are direct commands and arguments for the processor.

4.1. Data sets

A group of 30 data sets was developed for the purpose of our experiments. The sets were randomly generated based on experiments and industrial data. They were developed around two actual examples of scheduling productions for four weeks. The first example (with three products) came from the electronics industry (Kaczmarczyk, 2006), and the second one (with two products) – from the automotive industry (Miodońska, 2006).

All of the data sets had a number of periods T that was equal to 30 and a fixed period length C that was equal to 100. The tasks were diversified in terms of the numbers of products ($N = 5, 10, 15$) and machines ($\mu = 5, 10$). All of the data was randomly generated as integers by means of a uniform distribution U. The random parameters for each data set were as follows:

- The demand d_{jt} for product j in period t, was drawn from uniform distribution $U[1, 100]$ and reduced to 0 with a 0.4 probability; in addition, the demand in six initial scheduling periods was always 0 in order to ensure that there were acceptable solutions.
- Unit time p_j of manufacturing product j was randomly generated by means of a uniform distribution $U[1, 5]$.
- The unit cost h_j of product j's inventory was randomly generated by means of a uniform distribution $U[1, 5]$.
- The changeover time S_j^T for product j was randomly generated by means of a uniform distribution $U[0.2C, 0.8C]$ for equal production times of the machines in periods C.
- The changeover cost S_j^C of product $j$was randomly generated by means of a uniform distribution $U[10H_j, 150H_j]$, where $H_j = h_j C/p_j$ is equal to the total inventory cost of product j in a quantity that is equal to the daily production volume for the duration of one period.

The demand was drawn so that the level of the machine utilization was within a range of 75–90%. We assumed three changeovers for each product. Table 2 presents the average machine utilization levels without changeover times for the data sets that were used in the computational experiments.

Table 2. *Average machine-utilization level without changeover times for data sets that were used in computational experiments*

N/μ	5 [%]	10 [%]
5	79.8	82.2
10	86.0	75.0
15	80.0	87.8

N – number of products, μ – number of machines

4.2. Results of computational experiments

All of the calculations were made using a computer with an Intel COREi7 processor, 4710HQ, and 16 GB of RAM.

Solutions were sought with GUROBI 6.0.4 software with standard settings using the PLSP/F (1) model for the task of lot sizing and scheduling for identical parallel machines.

Heuristic solutions were found for all of the data sets by means of the heuristic BAPLSP/F algorithm for the same ranges of random parameters that were generated using a U-distribution or an integer distribution U^{INT}. The ranges of the random parameter values are shown in Table 3.

Table 3. *Value ranges for random parameters*

Parameter	Distribution	Value
γ	U	$[0.05, 0.95]$
δ	U	$[0.00, 30.00]$
ϵ	U	$[0.00, 1.00]$
ψ	U^{INT}	$[0, 3]$
σ	U^{INT}	$[1, 3]$
ω	U	$[0.00, 1.00]$
α	U	$[0.20, 1.00]$

U – uniform distribution, U^{INT} – integer uniform distribution

For each case, ten thousand runs of the BAPLSP/F heuristic algorithm were performed. From the acceptable solutions for the given task, the task with the best value of the target function was chosen as the solution.

The calculation time of ten thousand algorithm runs (Table 4) ranged from about 2 to 5 seconds depending on the size of the task. For all of the data sets, the search for optimal solutions was carried out with the GUROBI software. The calculation times were limited to 600 seconds each.

Let GAP determine the distance between the previously found acceptable integer solution and the best-known estimate for the task:

$$GAP = \left(\frac{|ObjBound - ObjVal|}{|ObjVal|} \right) 100\%, \tag{7}$$

where:

$ObjBound$ – best-known estimate for task,
$ObjVal$ – best-known solution for task.

Let G determine the relative distance between the obtained BAPLSP/F solution and the target function value of the solution that was computed with GUROBI:

$$G = \left(\frac{FC(BAPLSPF)}{FC(GUROBI)} - 1 \right) 100\%, \tag{8}$$

where:

$FC(BAPLSPF)$ – value of target function computed with new algorithm,
$FC(GUROBI)$ – value of target function computed with GUROBI software.

The average quality G of the solutions that were obtained using the BAPLSP/F algorithm for the particular data sets is shown in Table 4.

Table 4. *Results of computational experiments*

μ	N	G [%]	Time of calculations [s]		GAP [%]
			Heuristics	MIP	
5	5	4.75	1.89	3.96	0.00
	10	3.50	2.21	600.00	27.60
	15	–	3.31	600.00	–
10	5	0.99	2.41	1.23	0.00
	10	9.63	3.89	380.96	3.35
	15	–	4.78	600.00	–

μ – number of machines,
N – number of products,
MIP – by means of GUROBI solver.

For those sets with 15 products, it was not possible to calculate acceptable solutions for the PLSP/F task with the GUROBI solver. For those sets with five products (be it for five or ten machines), the GUROBI solver found the optimal solutions ($GAP = 0$). The values of the target functions in the solutions that were calculated using the BAPLSP/F algorithm were 4.71% worse on average than the solutions that were calculated using the GUROBI software in a maximum time of 600 seconds. The actual average calculation time for the new algorithm and the GUROBI solver showed that those sets with the smallest numbers of machines and with the smallest numbers of products had similar solution search times. As far as the remaining sets, the GUROBI software needed much more time to find a solution with a quality that was comparable to that of a heuristic solution (Table 4).

The heuristic algorithm computed the worst solution for the task with five products and five machines. The value of the target function was about 11% worse than could be found in the optimal solution. In another task with five products and five machines, a solution that was close to the optimal solution was found (the difference in the target function value being about 0.3%).

Figure 1 shows the average quality of the solutions G that were obtained with the BAPLSP/F algorithm as compared to the best-known solutions that were obtained with GUROBI (for the PLSP/F model). GUROBI needed 5 s, 10 s, 50 s, 100 s, and 600 s, respectively, to find a solution.

Those solutions that were computed with GUROBI in times that were comparable to the new BAPLSP/F algorithm were 24% worse on average than those that were obtained with the new heuristics. A doubled solution search time resulted in a less than 9% average improvement in the obtained solutions. The solutions that were found after 50 seconds were only about 4% worse than the heuristic solutions.

For the prepared data sets, the GUROBI software needed about 100 seconds to provide solutions that were similar in quality to those that were obtained by means of the new heuristic algorithm in less than 5 seconds. Extending the time of the solution search with the GUROBI software by more than 100 seconds allowed us to improve the quality of the obtained solutions by only a few percentage points on average.

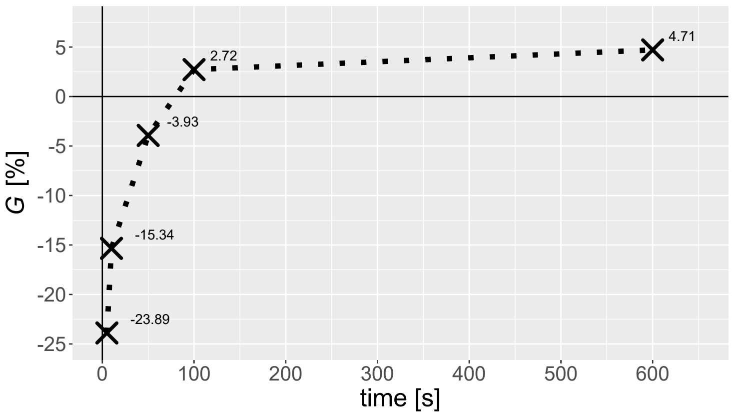


Fig. 1. *Average quality of solutions G found by GUROBI: 5 s, 10 s, 50 s, 100 s, and 600 s*

5. CONCLUSIONS

Lot sizing and scheduling plays an important role in the management of a production company. It is not always possible to use PLCM methods. As the PLCM models of lot sizing and scheduling are NP-hard, a small increase in a number of products significantly increases the time that is required to find a solution. In certain practical situations, waiting a long time for a good solution is not acceptable.

Therefore, new heuristic algorithms are constantly being developed for various lot-sizing and scheduling problems.

This paper describes a new algorithm for the task of lot-sizing and production scheduling with parallel machines. This algorithm is an adaptation of the heuristic BAPLSP algorithm that was proposed in 1994 by Hasse for a task with one machine.

The reduced time that is required to obtain a solution is an advantage of the new heuristic algorithm (BAPLSP/F). As a result, it is possible to run the BAPLSP/F algorithm to search for a solution many times. In such a case, the best solution is selected from a set of acceptable solutions. Each attempt to find a solution means a separate run of an algorithm; the algorithm does not "learn" during its operation.

As the computational experiments have shown, the new algorithm efficiently provides good solutions for tasks with large numbers of products and machines. Increased numbers of products only slightly extends the times that are required to find solutions for the tasks of lot sizing and scheduling with parallel machines. Therefore, the practical application of the new BAPLSP/F algorithm that is proposed in this paper has become possible.

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Decision Support for Allocating Farmed Fish to Customer Orders Using a Bi-objective Optimization Model

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Abstract. Aquaculture is an important industry in certain coastal areas. Focusing on the farming of salmon and trout, an operational planning problem arises with the goal of allocating a supply of fish to the demand that is expressed through customer orders. This paper provides a conceptual model of such a planning problem and defines a corresponding bi-objective mathematical programming model. The problem is novel with respect to the structure of fish transport and the rules for satisfying customer orders with respect to fish size, quality, certification, and health status. Computational experiments have been conducted to gain further insight into the use of the provided model to provide support for planners who are involved in operational decision-making. The results indicated that the bi-objective optimization model can be useful in situations where a supply is insufficient to cover all of the demand within a given planning horizon.

Keywords: augmented epsilon-constraint method, multi-objective model, fish supply chain, assignment problem, decision support

Mathematics Subject Classification: 90C29, 90B80

JEL Classification: Q22

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1. INTRODUCTION

The farming of salmon and trout in coastal areas has become an increasingly important food source, providing significant contributions to the economies of the producing countries (NOU, 2019). The aquaculture industry is subject to regulations regarding the maximum-allowed biomass and must address the health of the grown fish (Norwegian Food Safety Authority, 2020). In addition, exported fish may need certifications (Kiwa Norway, 2021).

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Figure 1 illustrates a typical supply chain for farmed fish. This paper addresses a planning problem that arises in the later parts of the chain. After the fish are slaughtered, they must be assigned to customer orders. These orders are either internal (as a company has its own processing facilities) or external (meaning that receiving customers are outside a company's facilities). The allocation to orders should maximize the amount of fish that is delivered while also fulfilling as many high-priority customer orders as possible.

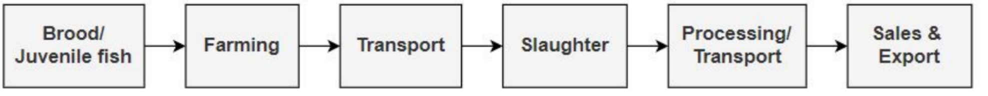

Fig. 1. *Supply chain in aquaculture*

Perishable products such as fish increase the complexity of supply chains because of their limited shelf-lives (Koldborg Jensen et al., 2010), by the fact that their values decrease while moving downstream (Musavi & Bozorgi-Amiri, 2017), and through the high variability of their price and demand (Ahumada & Villalobos, 2009). Koldborg Jensen et al. (2010) highlighted several characteristics in the supply chains of the fish industry. In general, different species of fish belong to different chains from the time of their catch to consumption, but some species may be interchangeable from the point of view of the end customer; this leads to an interdependence among the different chains. Aquaculture can occasionally be a parallel source to wild catch, but it can also result in independent supply chains from farm to fork.

At the upstream end of the supply chain, fish are caught along with the fish breeding in farms (Koldborg Jensen et al., 2010), while fresh and processed sales are at the downstream end. Between these, there are several agents who handle and process the fish and their products. Abedi and Zhu (2017) divided such supply chains that involve fish or other livestock into two parts; the warm chain, and the cold chain. The warm chain covers live fish, whereas the cold chain covers the products after harvest and processing. The latter is the focus of this paper.

Some related research has been conducted on the use of optimization models to support decisions in supply chains with perishable products. Ahumada and Villalobos (2011) presented a mixed-integer programming model to decide which agricultural products to harvest, how many times per week to harvest them, and on which days to harvest them. They also considered restrictions on time and labor and how harvest decisions affected the quality of the products. Amorim et al. (2012) developed a multi-objective mixed-integer programming model that covered the production and distribution of perishable goods. The focus was to minimize the total cost and maximize the mean remaining shelf life. Musavi and Bozorgi-Amiri (2017) considered a multi-objective optimization problem for a perishable food supply chain. They applied a heuristic solution method (NSGA-II) to generate an approximation of the Pareto front.

Abedi and Zhu (2017) provided a mixed-integer programming model to maximize the profit of a trout supply chain. The output of the model included the purchase quantity (of trout spawns), a harvest plan, and a distribution plan. The distribution plan also involved customer prioritization based on quantity in demand in order to efficiently find a way to deliver fish. The authors remarked that, up until that point in time, fish-farming companies had not taken much advantage of efficient distribution planning. Mosallanezhad et al. (2021) investigated a shrimp supply chain network, focusing on the locations of facilities such as distribution centers, wholesalers, factories, and markets as well as determining the flow of the products and waste within the network.

Outside of supply chains for perishable products, another stream of relevant research focuses on order allocation. This includes research on combined supplier selection and order allocation (Jia et al., 2020; Kaviani et al., 2020; Moheb-Alizadeh & Handfield, 2019; Sharma & Darbari, 2021). However, the existing literature considers allocating orders to suppliers from the point of view of the producer, whereas our research considers customer orders. Another direction of this type of research is the combination of location and allocation; for example, regarding nursing homes (Wang & Ma, 2018) or crisis situations (Ghasemi et al., 2019) (where one first locates facilities and then allocates flow).

Fan et al. (2019) tackled orders from customers at a brand manufacturer where the orders were aggregated before the brand manufacturer sent them to the equipment manufacturers. They had a multi product-period-equipment manufacturer problem and presented an integer nonlinear programming formulation. To solve their problem, a novel genetic algorithm was developed. Seitz et al. (2020) allocated supply to customer orders (as in our work) while taking into account the fact that the demand was forecast (which differed from our setting). In general, there is less research on order allocation under uncertainty; however, there are some examples (such as the allocation of uncertain customer orders to machines (Zhang et al., 2022)).

In our problem, we considered two objectives: maximizing deliveries, and the number of high-priority orders that are fulfilled. The ε-constraint method, the augmented ε-constraint method, or variants of these have been used to solve optimization models for many different types of supply chains, including the dairy industry (Gholizadeh et al., 2021), oil and gas (Ebrahimi & Bagheri, 2022), waste management (Abdollahi Saadatlu et al., 2022; Rabbani et al., 2020), nursing homes (Wang & Ma, 2018), and evacuation planning (Ghasemi et al., 2019). In a relevant article, Fasihi et al. (2021b) proposed a novel mathematical model for minimizing the cost of a closed-loop supply chain for fish.

The ε-constraint method tends to be time-consuming and not always able to solve all instances within a reasonable time limit; therefore, it is often compared to heuristic solution methods (Fasihi et al., 2021a). Ghasemi et al. (2019) found that the ε-constraint method provided good results (despite being slow), and Ebrahimi and Bagheri (2022) preferred the ε-constraint method over goal programming.

Our research is motivated by companies that farm, slaughter, and sell fish. The companies have planners who allocate slaughtered fish to customer orders; the combination of the relevant aspects of this planning problem has not been studied in the academic literature. This includes how fish transport is conducted and the rules for fulfilling orders with respect to fish quality, size, certification, and health status. This paper contributes to the literature by providing a conceptual model of the problem at hand and then producing a mixed-integer programming model for the problem. Finally, we report on computational experiments that were performed to assess the usefulness of the mathematical model. Based on the results, we aim to characterize situations where bi-objective optimization is beneficial and, on the other hand, situations where single-objective optimization is sufficient.

The remainder of this paper is structured as follows. Section 2 describes the problem that was modeled and solved. Next, Section 3 presents a mixed-integer programming model for the problem. A computational study to evaluate the model and the obtained solutions is contained in Section 4, followed by our conclusions in Section 5.

2. PROBLEM DESCRIPTION

When fish is slaughtered, planners need to assign the fish to any pending customer orders; after this, the fish is transported to the customers. We focus on the decisions that planners must make when assigning fish to orders, the information that is available when making decisions, and the restrictions to which the decisions must adhere. The plans are made before the start of each week; then, the planning horizon is Monday through Friday (as illustrated in Figure 2).

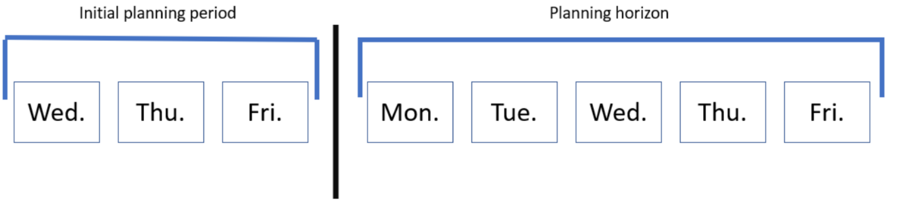

Fig. 2. *Initial planning period and planning horizon*

We next describe the physical flow of fish through the supply chain, followed by the flow of information with respect to the planning process and further details regarding the process of allocating the fish to the customers' orders.

2.1. Physical flow

The main entities that were considered in this research were fish farms, plants, and distribution centers. The physical flow of fish from farms is illustrated in Figure 3, which follows the flow from the start of an order to the end customer.

Fish farms are located along coasts and consist of facilities where fish are grown. At the time of harvest, the fish are loaded onto a well boat and transported from the farm to the plants. A plant consists of a slaughterhouse, a packing facility, and possibly a processing facility that is located in the same building. When arriving at the plant, the fish are unloaded and delivered to the slaughterhouse. When the slaughter process is completed, the fish are either forwarded to an internal-processing facility or transported out of the plant.

If the next step is the internal-processing facility, the fish are sent directly from the slaughterhouse to the facility and further processed into filets or other fish products. Otherwise, the fish must first be packed into boxes at the packing facility. A standard box weighs around 20 kg when fully packed. After being packed, the boxes are ready for storage or further transport. The storage capacity in a plant is limited, and the boxes are only stored there for a short period of time. When the boxes are shipped from the plant, the next destination can be a final customer or a distribution center.

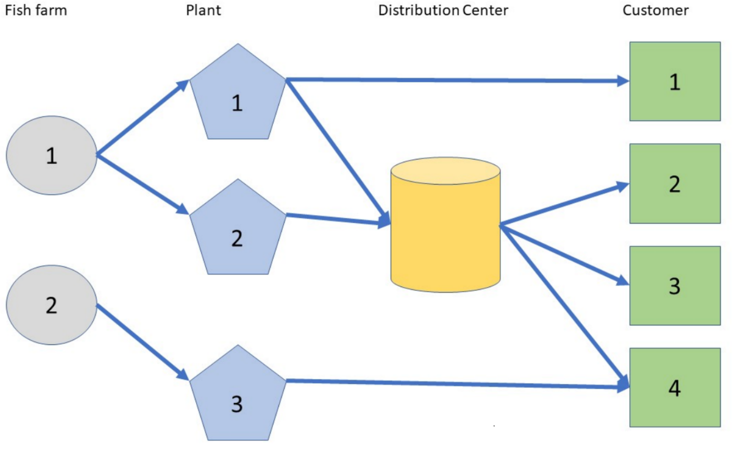

Fig. 3. *Simplified illustration of physical flow*

A distribution center is used as an intermediary hub and has the features of consolidation and storage. All of the incoming supply to the distribution center comes from the plants. There is a certain distance between a plant and a distribution center, and the lead time is one or two days. If necessary, it is possible to consolidate the supply from multiple plants prior to further transport. The distribution center has a significantly larger inventory capacity than the plants do, and long-term storage is allowed so that boxes can be stored up until the expiration dates of the fish.

2.2. Information flow

Fish are characterized by three attributes: species, quality, and size class. There are two species involved: salmon, and trout. Each is split into three different qualities and then subdivided into nine different size classes based on the weights of the fish.

A fish farm has a set of certifications and a given status for the health of the fish. The fish that comes from a given farm is associated with the same certificates and health status. When the fish have grown to an acceptable weight and size, they are ready for harvest. The fish are harvested on a specific day and sent directly to a plant for further processing. The planners know the day and time that the fish will arrive at the plant but cannot influence this time themselves.

The planners also have access to continuously updated forecasts of the amounts of fish that will be delivered to a plant. The species of the incoming fish is known, but the forecast has more-specific data on the total amount of fish, the amount of the fish from each size class, and the amount of the fish with a given quality. When the fish is slaughtered, it is also weighed and its quality checked. At this point, the planners will begin to receive more-accurate information about the available amounts of the supplies.

The demand for fish is expressed through orders. The main elements of an order are details about the requested species along with the size class, quality, number of boxes, certification, health status, delivery date, and delivery location. A sales department works closely with planners and handles all orders, customer relationships, contracts, and prices. In cooperation, the planners and sales department decide about the priorities of the individual orders.

An order contains details about the numbers of boxes that are requested for a given combination of species, size class, and quality. The orders can allow for flexibility in the numbers of boxes that are delivered for a given combination, which are given as upper and lower bounds on the numbers of boxes. If it is possible to deliver within the bounds, the order is considered to be fulfilled. While an order specifies a certain species and quality, it may allow for the delivery of multiple size classes.

Each order specifies the date on which the customer wants the fish to reach its destination. When allocating fish to orders, the planners use the day of delivery to decide when the supply should be sent from the plant. An order can allow for some flexibility with respect to the requested delivery date.

The orders can specify that the fish must have one or more certifications. The customers can also refrain from receiving fish with specific health statuses. The federal governments of certain countries have their own requirements for the health and certification of fish; this means that, if the fish is to be exported to such countries, then it needs to meet their specific requirements.

An important distinction is between external and internal orders. An internal order is an order that comes from the internal-processing facility at a plant. External orders have destinations that are outside the plants, which require the transportation of the fish from the plants to the final destinations (possibly via distribution centers).

2.3. Allocation process

The allocation process has the goal of allocating supply to demand in the best possible way. The planners go through five defined steps when creating a plan:

1) receiving orders,
2) checking orders,
3) prioritizing orders,
4) checking whether it is possible to fulfill orders,
5) delivering or canceling orders.

Planners receive order details from the sales department and check what is requested. The orders are then assigned their priorities based on contractual obligations, customer relationships, and prices. Sometimes, a customer must deal with deviations from their specified demand. The planners aim to deliver to customers with as little deviation as possible over a longer period of time. Some customers place regular orders, which opens up the possibility of leveling out the deviations over time. If a customer has a significant deviation from their demand in one week, their order for the following week can be set to a higher priority.

The available supply is known exactly or is estimated. Based on the above information, the planners can start the allocation of supply to demand while deciding how each order is to be fulfilled. If it is possible to fulfill an order, it can be accepted; otherwise, it must be cancelled. When all of the orders that can be fulfilled are processed, there might be unsold fish left. The sales department then tries to obtain additional orders (for example, in the spot market) so that all of the fish can be sold. If this is impossible, the fish is stored until the next planning horizon (usually at a distribution center).

In those weeks when there is an insufficient supply, the planners are forced to decide which orders to fulfill. In those weeks with surpluses of supply, however, challenges arise regarding where to store the boxes. The decision about finding appropriate locations to store the boxes is affected by the storage costs, the available inventory capacity, and the shelf life of the fish.

Two goals are studied in this paper. The first goal is to provide as much of the available supply as possible to orders. This will lower the total number of unsold boxes that are left at the end of a planning horizon. Lowering the numbers of remaining unsold boxes can potentially lower storage costs. The second goal is to fulfill as many highly prioritized orders as possible. As previously stated, the priority of an order is based on contracts, customer relationships, and prices. Satisfying this goal will please important customers and can increase profits. The interesting connection between the two goals is that they can be conflicting or nonconflicting depending on the balance among the supply, the demand, and the prioritization of the orders.

3. MATHEMATICAL MODEL

The following model intends to capture the fundamental essence of the fish-allocation process. It is designed for use just before weekly operations start and is not meant for dynamic replanning during a planning horizon.

The fish are reared in individual farms; L denotes the set of all farms. Outbreaks of disease can occur on any farm; the set of possible diseases is written as E. The farms can be certified; the set of possible certifications is indicated by H.

After harvesting the fish from a farm, it is sent to a plant for slaughter and packing; A denotes the set of all plants. The fish are described using the following attributes: species, size class, and quality. To this end, we introduce the set S of the species, the set Z of the size classes, and the set Q of the possible qualities.

The model is designed for a seven-day schedule, but the notation is kept flexible. All of the plants are open from day 1 through day T. The lead time from plant a to a distribution center is R_a, which is at least one full day. Parameter F represents the longest lead time from a plant to a distribution center, with $F = max_{a \in A} \{R_a\}$. All of the supply that is sent from a plant on day T must arrive at a distribution center before a planning horizon ends. Therefore, the length of the planning horizon is $T + F$, and the distribution center is open from day 1 through day $T + F$.

There are two types of orders. External orders have destinations that are outside a plant and are elements of set O^{EX}. Internal orders are related to the processing facilities that are located in the same buildings as the plants and are elements of set O^{IN}. Parameter K_o is used to indicate the importance of an order o, with higher values indicating a higher priority when serving the orders.

External orders have two different methods of delivery, while internal orders have only one. Direct delivery is defined as sending boxes directly from one plant to a customer without the need to consolidate from other plants. Variable $x^A_{l,a,s,z,q,t,o}$ represents the number of boxes that contain fish from species s, size class z, and quality q that are harvested on farm l and sent directly from plant a on day t to cover order o. Binary variable $y^A_{a,t,o}$ takes a value of 1 if order o is delivered directly from plant a on day t; otherwise, this value is 0.

If the delivery of an order is made from a distribution center, there cannot also be a direct delivery from a plant to the same order. Variable $x^{DC}_{l,a,s,z,q,t,o}$ represents the number of boxes that contain fish from species s, size class z, and quality q that are harvested on farm l and sent from plant a through a distribution center to order o on day t. Binary variable $y^{DC}_{t,o}$ takes a value of 1 if order o is served from a distribution center on day t; otherwise, this value is 0.

Internal orders have only one method of delivery, which is in-house delivery. It is not allowed to serve an internal-processing facility in plant a from a distribution center or other plants. Variable $x^{IN}_{l,a,s,z,q,t,o}$ represents the number of boxes that contain fish from species s, size class z, and quality q that are harvested on farm l and delivered to the internal-processing facility of plant a to deliver order o on day t. Binary variable $y^{IN}_{t,o}$ takes a value of 1 if internal order o is served on day t; otherwise, the value is 0.

Parameters:

- T – number of days that plant is open,
- F – longest lead time from plant to distribution center,
- $P_{l,a,s,z,q,t}$ – supply coming from farm l to plant a of species s, size class z, and quality q on day t,
- $D^{TOTAL}_{s,q,o}$ – total demand of boxes of fish of species s and quality q in order o,
- $D^{MAX}_{s,z,q,o}$ – maximum demand of boxes of fish of species s, size class z, and quality q in order o,
- $D^{MIN}_{s,z,q,o}$ – minimum demand of boxes of fish of species s, size class z, and quality q in order o,
- K_o – importance of order o,
- G_o – relative lower bound on fish delivered to order o of given species and quality,
- $\sigma^A_{t,o}$ – 1 if order o does not allow delivery on day t, 0 otherwise,
- $\sigma^{DC}_{t,o}$ – 1 if order o does not allow delivery on day t, 0 otherwise,
- $\sigma^{IN}_{t,o}$ – 1 if order o does not allow delivery on day t, 0 otherwise,
- $I^A_{l,a,s,z,q}$ – initial number of boxes of fish of species s, size class z, and quality q from farm l in inventory at plant a,
- $I^{DC}_{l,a,s,z,q}$ – initial number of boxes of fish of species s, size class z, and quality q coming from farm l through plant a in inventory at distribution center,

C_a^A – inventory capacity at plant a,
R_a – lead time from plant a to distribution center,
$\delta_{a,o}^A$ – 1 if plant a can deliver to order o, 0 otherwise,
$U_{o,e}^O$ – 1 if order o does not accept fish from farm with disease e, 0 otherwise,
$U_{l,e}^L$ – 1 if fish comes from farm l without disease e, 0 otherwise,
$\pi_{o,h}^O$ – 1 if order o requires fish with certificate h, 0 otherwise,
$\pi_{l,h}^L$ – 1 if fish from farm l have certificate h, 0 otherwise,
J^A – weight of making direct delivery,
J^{DC} – weight of making delivery through distribution center,
M – large constant used to ensure linear constraints.

Variables:

$x_{l,a,s,z,q,t,o}^A$ – number of boxes of fish of species s, size class z, and quality q from farm l sent directly from plant a to order o on day t,
$x_{l,a,s,z,q,t,o}^{DC}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a through distribution center to order o on day t,
$x_{l,a,s,z,q,t,o}^{IN}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a to internal order o on day t,
$y_{a,t,o}^A$ – 1 if delivery to order o on day t directly from plant a is done, 0 otherwise,
$y_{t,o}^{DC}$ – 1 if delivery to order o on day t directly from distribution center is done, 0 otherwise,
$y_{t,o}^{IN}$ – 1 if delivery to internal order o on day t from plant is done, 0 otherwise,
$\mu_{l,a,s,z,q,t}^A$ – number of boxes of fish of species s, size class z, and quality q from farm l stored at plant a on day t,
$\mu_{l,a,s,z,q,t}^{DC}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a stored at distribution center on day t,
$b_{l,a,s,z,q,t}$ – number of boxes of fish of species s, size class z, and quality q from farm l sent from plant a to distribution center on day t.

3.1. Objective functions

We define two objective functions ($W1$ and $W2$) as representing the total number of boxes that have been delivered to all orders and the total value of the prioritized orders that have been fulfilled, respectively. Both objective functions are to be maximized.

$$\max W1 = \sum_{l\in L}\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T}\sum_{o\in O}^{\varepsilon x} J^A x_{l,a,s,z,q,t,o}^A + \right.$$
$$\left. + \textstyle\sum_{t=1}^{T+F} J^{DC} x_{l,a,s,z,q,t,o}^{DC} + \sum_{t=1}^{T}\sum_{o\in O^{IN}} x_{l,a,s,z,q,t,o}^{IN}\right). \tag{1}$$

The first goal is to deliver as much as possible of the available supply to the orders, which corresponds to maximizing the total number of boxes that are delivered to all orders. It is possible for planners to predefine which delivery method is preferred for any external orders. Weights for direct delivery J^A and delivery through distribution center J^{DC} are multiplied with their respective variables. A relatively higher weight on one of the two parameters indicates which of the delivery methods is preferred; each weight is a positive number that is less than or equal to 1.

$$\max W_2 = \sum_{o \in O^{\varepsilon x}} K_o \left(\sum_{a \in A} \sum_{t=1}^{T} y^A_{a,t,o} + \sum_{t=1}^{T+F} y^{DC}_{t,o} \right) + \sum_{o \in O^{IN}} K_o \sum_{t=1}^{T} y^{IN}_{t,o} . \tag{2}$$

The second goal is to fulfill as many highly prioritized orders as possible; that is, maximizing the total value of the prioritized orders that have been fulfilled.

3.2. Constraints

Initial inventory

$$\mu^A_{l,a,s,z,q,t} = I^A_{l,a,s,z,q}, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q, t = 0 , \tag{3}$$

$$\mu^{DC}_{l,a,s,z,q,t} = I^{DC}_{l,a,s,z,q}, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q, t = 0 . \tag{4}$$

The first set of constraints governs the initial inventories; Constraints (3) make sure that the initial inventory is set in a plant, while Constraints (4) handle the initial inventory in a similar fashion in a distribution center.

Inventory capacity constraints at plant

$$\sum_{l \in L} \sum_{s \in S} \sum_{z \in Z} \sum_{q \in Q} \mu^A_{l,a,s,z,q,t} \le C^A_a , \quad a \in A,\ t \in \{0, \ldots, T\} . \tag{5}$$

The value of variables $\mu^A_{l,a,s,z,q,t}$ must be less than or equal to the inventory capacity at plant a, as is enforced by Constraints (5). The distribution center has no inventory-capacity constraint due to the assumption that its capacity is infinite.

Balance constraints at plants

$$\begin{gathered} P_{l,a,s,z,q,t} + \mu^A_{l,a,s,z,q,t-1} - \sum_{o \in O^{\varepsilon x}} x^A_{l,a,s,z,q,t,o} - \\ - \textstyle\sum_{o \in O^{IN}} x^{IN}_{l,a,s,z,q,t,o} - b_{l,a,s,z,q,t} = \mu^A_{l,a,s,z,q,t}, \\ l \in L, a \in A, s \in S, z \in Z, q \in Q, t = 0 . \end{gathered} \tag{6}$$

Constraints (6) balance the inventory and the incoming and outgoing flows for plant a. The available supply in period t consists of incoming supply $P_{l,a,s,z,q,t}$ and the inventory from previous period $\mu^A_{l,a,s,z,q,t-1}$. If there is no available supply for a given combination of farm, plan, species, size class, and quality, all of the variables with this combination are set to 0. Variables $x^A_{l,a,s,z,q,t,o}$ and $x^{IN}_{l,a,s,z,q,t,o}$ state the number of boxes delivered directly or in-house, respectively. Variables $b_{l,a,s,z,q,t}$ represent the number of boxes that are sent from plant a to a distribution center. Sending boxes from a plant to a distribution center is done in order to ensure that there are enough boxes that are available for further delivery or to store the boxes at the distribution center instead of at the plant. All of the boxes that are left on day t at plant a are stored for the next period.

Balance constraints at distribution center

$$\begin{gathered}\mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o \in O^{\varepsilon x}} x^{DC}_{l,a,s,z,q,t,o} = \mu^{DC}_{l,a,s,z,q,t}, \\ l \in L, a \in A, s \in S, z \in Z, q \in Q, t \in \{1, \dots, R_a\}\,,\end{gathered} \tag{7}$$

$$\begin{gathered}b_{l,a,s,z,q,t-R_a} + \mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o \in O^{\varepsilon x}} x^{DC}_{l,a,s,z,q,t} = \mu^{DC}_{l,a,s,z,q,t}, \\ l \in L, a \in A, s \in S, z \in Z, q \in Q, t \in \{1 + R_a, \dots, T + R_a\}\,,\end{gathered} \tag{8}$$

$$\begin{gathered}\mu^{DC}_{l,a,s,z,q,t-1} - \sum_{o \in O^{\varepsilon x}} x^{DC}_{l,a,s,z,q,t,o} = \mu^{DC}_{l,a,s,z,q,t}, \\ l \in L, a \in A, s \in S, z \in Z, q \in Q, t \in \{T + R_a + 1, \dots, T + F\}\,.\end{gathered} \tag{9}$$

Three sets of constraints are used to force the correct balance of inventory, incoming flow, and outgoing flow, respectively, at a distribution center. All of the incoming flow to the distribution center comes from the plants. All of the outgoing flow is the number of boxes that are delivered from the distribution center to the orders.

The lead time from plant a to a distribution center is R_a, which means that it takes R_a days to send the supply from plant a to the distribution center. The distribution center is open from Day 1, but the incoming supply can only arrive R_a days after leaving plant a. This affects the incoming flow at the distribution center. Constraints (7) ensure that there is balance in inventory and out-going flow in the periods before new supply can arrive from plant a.

The first day that the supply that is sent from plant a can arrive at a distribution center is on day $1 + R_a$. Constraints (8) ensure the balance of inventory, incoming flow, and outgoing flow at the distribution center from period $1 + R_a$ through $T + R_a$. Variables $b_{l,a,s,z,q,t-R_a}$ represent the number of boxes that are sent on day $t - R_a$ from plant a and arrive on day t at the distribution center.

The distribution center is open in Period 1 through $T + F$, but the constraint set (8) is only valid from $1 + R_a$ to $T + R_a$. If R_a is less than F for plant a, then this causes a problem. Variables $\mu^{DC}_{l,a,s,z,q,t-1}$, $x^{DC}_{l,a,s,z,q,o}$, and $\mu^{DC}_{l,a,s,z,q,t}$ that are related to plant a are unbounded in the periods after $T + R_a$. Constraints (9) address this problem and ensure that, from $T + R_a + 1$ through $T + F$, all of the variables that are related to those plants with lead times that are shorter than F are bounded.

Variables $b_{l,a,s,z,q,t-R_a}$ are omitted from Constraints (9). An example is used to explain why the variables are omitted. If the lead time of plant a is one day and it is now day $T + 2$, then variables $b_{l,a,s,z,q,t-R_a}$ are related to the number of boxes that left plant a on day $T + 1$. The plant is only open from day 1 through day T, so no boxes leave from plant a after day T. It is therefore redundant to have variables that are related to the number of boxes that are sent on day $T + 1$. This is the reason for not including $b_{l,a,s,z,q,t-R_a}$ in the constraint set.

Delivery

$$\sum_{a \in A} \sum_{t=1}^{T} y^{A}_{a,t,o} + \sum_{t=1}^{T+F} y^{DC}_{t,o} \leq 1, \quad o \in O^{\varepsilon x}, \tag{10}$$

$$\sum_{t=1}^{T} y^{IN}_{t,o} \leq 1, \quad o \in O^{\varepsilon x}. \tag{11}$$

Constraints (10) make sure that, if an external order is accepted, only one of the two available delivery methods for external orders is used. If the direct delivery from a plant is used, then only one plant can deliver to the order. It is only allowed to deliver one time to an order throughout the entire planning horizon regardless of the chosen delivery method. If an internal order is accepted, it is only served once throughout the planning horizon, as is indicated by Constraints (11).

Day of delivery

$$\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q} x^{A}_{l,a,s,z,q,t,o} \le M(1-\sigma^{A}_{t,o}), \quad t\in\{1,\ldots,T\}, o\in O^{\varepsilon x}, \tag{12}$$

$$\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q} x^{DC}_{l,a,s,z,q,t,o} \le M(1-\sigma^{DC}_{t,o}), \quad t\in\{1,\ldots,T+F\}, o\in O^{\varepsilon x}, \tag{13}$$

$$\sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q} x^{IN}_{l,a,s,z,q,t,o} \le M(1-\sigma^{IN}_{t,o}), \quad t\in\{1,\ldots,T\}, o\in O^{IN}. \tag{14}$$

The day of delivery is the day that the supply must be sent so it can arrive on the day that is specified in the order. Constraints (12) ensure that it is only possible to deliver directly from a plant to an external order on the allowed day of delivery. Similarly, Constraints (13) state that it is only possible to deliver from the distribution center to an external order on the allowed day of delivery. Finally, Constraints (14) ensure that it is only possible to deliver to an internal order on the allowed day of delivery.

Demand – upper bound

$$\begin{gathered}\sum_{l\in L}\sum_{z\in Z} x^{A}_{l,a,s,z,q,t,o} \le D^{TOTAL}_{s,q,o}\, y^{A}_{a,t,o},\\ a\in A, s\in S, q\in Q,\ t\in\{1,\ldots,T\}, o\in O^{\varepsilon x},\end{gathered} \tag{15}$$

$$\begin{gathered}\sum_{l\in L}\sum_{a\in A}\sum_{z\in Z} x^{DC}_{l,a,s,z,q,t,o} \le D^{TOTAL}_{s,q,o}\, y^{DC}_{t,o},\\ s\in S, q\in Q,\ t\in\{1,\ldots,T+F\}, o\in O^{\varepsilon x},\end{gathered} \tag{16}$$

$$\begin{gathered}\sum_{l\in L}\sum_{a\in A}\sum_{z\in Z} x^{IN}_{l,a,s,z,q,t,o} \le D^{TOTAL}_{s,q,o}\, y^{IN}_{t,o},\\ s\in S, q\in Q,\ t\in\{1,\ldots,T\}, o\in O^{IN}.\end{gathered} \tag{17}$$

The model allows an order to have flexibility in the number of boxes that are delivered and the possibility of substituting the size classes that are used in the delivery. Flexibility is allowed in the number of boxes that are specified in an order. Parameter $D^{TOTAL}_{s,q,o}$ represents the demand for a specific combination of species s and quality q for order o. Three sets of constraints are related to separate methods of delivery and the upper bound on this demand, effectively stating that the deliveries to an order cannot exceed the maximum amounts of fish of a given species and quality levels that are requested.

The number of boxes that are delivered directly from plant a are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s and quality q; this controlled by Constraints (15). Constraints (16) ensure that the number of boxes that are delivered from the distribution center are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s and quality q, while Constraints (17) ensure that the number of boxes that are delivered in-house are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s and quality q.

Demand – lower bound

$$\begin{aligned}&\sum_{l\in L}\sum_{z\in Z} x^{A}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{A}_{a,t,o},\\ &a\in A, s\in S, q\in Q,\ t\in\{1,\ldots,T\}, o\in O^{\varepsilon x},\end{aligned} \tag{18}$$

$$\begin{aligned}&\sum_{l\in L}\sum_{a\in A}\sum_{z\in Z} x^{DC}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{DC}_{t,o},\\ &s\in S, q\in Q,\ t\in\{1,\ldots,T+F\}, o\in O^{\varepsilon x},\end{aligned} \tag{19}$$

$$\begin{aligned}&\sum_{l\in L}\sum_{a\in A}\sum_{z\in Z} x^{IN}_{l,a,s,z,q,t,o} \geq D^{TOTAL}_{s,q,o} G_o y^{IN}_{t,o},\\ &s\in S, q\in Q,\ t\in\{1,\ldots,T\}, o\in O^{IN}.\end{aligned} \tag{20}$$

Parameter G_o is a value that is within a range of 0 to 1. If parameter $D^{TOTAL}_{s,q,o}$ is multiplied by G_o, the lower bound on the demand for a specific combination of species s and quality q is found for order o. Constraints (18) force the number of boxes that are delivered directly from plant a to be more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s and quality q. Constraints (19) ensure that the number of boxes that are delivered from the distribution center are more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s and quality q. The number of boxes that are delivered in-house must be more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s and quality q; this is expressed by Constraints (20).

Demand – upper bound (size class)

$$\begin{aligned}&\sum_{l\in L} x^{A}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o}\, y^{A}_{a,t,o},\\ &a\in A, s\in S, z\in Z, q\in Q,\ t\in\{1,\ldots,T\}, o\in O^{\varepsilon x},\end{aligned} \tag{21}$$

$$\begin{aligned}&\sum_{l\in L}\sum_{a\in A} x^{DC}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o}\, y^{DC}_{t,o},\\ &s\in S, z\in Z, q\in Q,\ t\in\{1,\ldots,T+F\}, o\in O^{\varepsilon x},\end{aligned} \tag{22}$$

$$\begin{aligned}&\sum_{l\in L}\sum_{a\in A} x^{IN}_{l,a,s,z,q,t,o} \leq D^{MAX}_{s,z,q,o}\, y^{IN}_{t,o},\\ &s\in S, z\in Z, q\in Q,\ t\in\{1,\ldots,T\}, o\in O^{IN}.\end{aligned} \tag{23}$$

Parameter $D^{MAX}_{s,z,q,o}$ represents the upper bound on a given size class z that can be delivered with a combination of species s and quality q for order o. The parameter must be set to less than or equal to $D^{TOTAL}_{s,q,o}$. If $D^{MAX}_{s,z,q,o}$ is set to zero, it is not allowed to deliver anything of this size class z for this combination of species s and quality q for order o. If an order is accepted, Constraints (18)–(20) control the total amount that is delivered for the given species and quality of the fish. Combined with Constraints (21)–(26), a company has some flexibility in terms of which size classes are used to meet the demand.

Constraints (21) state that the number of boxes of size class z that are delivered directly from plant a must be fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q. Next, Constraints (22) ensure that the number of boxes of size class z that are delivered from the distribution center are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q. Then, Constraints (23) ensure that the number of boxes of size class z that are delivered internally are fewer than or equal to the maximum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q.

Demand – lower bound (size class)

$$\sum_{l\in L} x^{A}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o}\, y^{A}_{a,t,o}, \quad a\in A, s\in S, z\in Z, q\in Q,\ t\in\{1,\dots,T\}, o\in O^{\varepsilon x}, \tag{24}$$

$$\sum_{l\in L}\sum_{a\in A} x^{DC}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o}\, y^{DC}_{t,o}, \quad s\in S, z\in Z, q\in Q,\ t\in\{1,\dots,T+F\}, o\in O^{\varepsilon x}, \tag{25}$$

$$\sum_{l\in L}\sum_{a\in A} x^{IN}_{l,a,s,z,q,t,o} \geq D^{MIN}_{s,z,q,o}\, y^{IN}_{t,o}, \quad s\in S, z\in Z, q\in Q,\ t\in\{1,\dots,T\}, o\in O^{IN}. \tag{26}$$

Parameter $D^{MIN}_{s,z,q,o}$ represents the lower bound of the boxes of a given size class s that must be delivered with a combination of species s and quality q for order o. The value of parameter $D^{MIN}_{s,z,q,o}$ must be less than or equal to $D^{MAX}_{s,z,q,o}$. Then, Constraints (24) enforce that the number of boxes of size class z that are delivered directly from plant a are more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q.

Constraints (25) ensure that the number of boxes of size class z that are delivered from a distribution center are more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q. For internal orders, Constraints (26) state that the number of boxes of size class z that are delivered in-house must be more than or equal to the minimum number of boxes that are specified in order o for a given combination of species s, size class z, and quality q.

Disease

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T} x^{A}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F} x^{DC}_{l,a,s,z,q,t,o}\right) \leq M\left((1-U^{O}_{o,e}) + U^{L}_{l,e}\right), \quad L\in L, o\in O^{\varepsilon x}, e\in E, \tag{27}$$

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o}\right) \leq M\left((1-U^{O}_{o,e}) + U^{L}_{l,e}\right), \quad L\in L, o\in O^{IN}, e\in E. \tag{28}$$

If external order o is to receive fish that was grown in farm l, then Constraints (27) ensure that the health status of disease e is what is specified in the order. Similarly, if internal order o is to receive fish that was bred in farm l, Constraints (28) ensure that the health status of disease e is what is specified in the order.

Certifications

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T} x^{A}_{l,a,s,z,q,t,o} + \sum_{t=1}^{T+F} x^{DC}_{l,a,s,z,q,t,o}\right) \leq M\left((1-\pi^{O}_{o,h}) + \pi^{L}_{l,h}\right), \quad L \in L, o \in O^{\varepsilon x}, h \in H, \tag{29}$$

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o}\right) \leq M\left((1-\pi^{O}_{o,h}) + \pi^{L}_{l,h}\right), \quad L \in L, o \in O^{IN}, h \in H. \tag{30}$$

If external order o is to receive fish that was raised in farm l, then Constraints (29) force this fish to meet the certification requirements h that are specified in the order. Constraints (30) deal with internal orders in a similar way: if internal order o is to receive fish that was bred in farm l, then this fish must meet the corresponding requirements.

Internal-processing facility

$$\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\sum_{t=1}^{T} x^{IN}_{l,a,s,z,q,t,o} \leq M\delta^{A}_{a,o}, \quad a \in A, o \in O^{IN}. \tag{31}$$

An internal order is related to only one plant. Constraints (31) are used to enforce that only the plant that is related to order o is allowed to deliver fish. If $\delta^{A}_{a,o}$ is 1, then order o is related to plant a; otherwise, this value is 0.

Domain of variables

$$x^{A}_{l,a,s,z,q,t,o} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1,\ldots,T\}, o \in O^{\varepsilon x}, \tag{32}$$

$$x^{DC}_{l,a,s,z,q,t,o} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1,\ldots,T+F\}, o \in O^{\varepsilon x}, \tag{33}$$

$$x^{IN}_{l,a,s,z,q,t,o} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1,\ldots,T\}, o \in O^{IN}, \tag{34}$$

$$y^{A}_{a,t,o} \in \{0,1\}, \quad a \in A, t \in \{1,\ldots,T\}\, o \in O^{\varepsilon x}, \tag{35}$$

$$y^{DC}_{t,o} \in \{0,1\}, \quad t \in \{1,\ldots,T+F\}\, o \in O^{\varepsilon x}, \tag{36}$$

$$y_{t,o}^{IN} \in \{0,1\}, \quad t \in \{1,\ldots,T\}\, o \in O^{IN}, \tag{37}$$

$$\mu_{l,a,s,z,q,t}^{A} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1,\ldots,T\}, \tag{38}$$

$$\mu_{l,a,s,z,q,t}^{DC} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1,\ldots,T+F\}, \tag{39}$$

$$b_{l,a,s,z,q,t} \geq 0, \quad l \in L, a \in A, s \in S, z \in Z, q \in Q,\ t \in \{1,\ldots,TF\}. \tag{40}$$

All of the variables are non-negative.

3.3. Augmented ε-constraint method

The augmented ε-constraint method (Mavrotas, 2009) is used to solve a model with both objectives. First, a pay-off table is calculated in which each objective function is optimized individually, providing the best possible value for each objective function. The table also provides the worst value for each objective function, since the other objective functions are prioritized first according to the lexicographic method (Hwang & Masud, 1979). Let parameter r_k be the range of objective function k in the pay-off table.

In our implementation of the augmented ε-constraint method, the objective function that maximizes the total number of boxes that are delivered is made the primary objective:

$$\max \sum_{l\in L}\sum_{a\in A}\sum_{s\in S}\sum_{z\in Z}\sum_{q\in Q}\left(\sum_{t=1}^{T}\sum_{o\in O^{\varepsilon x}} J^{A} x_{l,a,s,z,q,t,o}^{A} + \sum_{t=1}^{T+F}\sum_{o\in O^{\varepsilon x}} J^{DC} x_{l,a,s,z,q,t,o}^{DC} + \sum_{t=1}^{T}\sum_{o\in O^{IN}} x_{l,a,s,z,q,t,o}^{IN}\right) + \lambda\left(\frac{S_2}{r_2}\right), \tag{41}$$

where S_k are the slack or surplus values, and the parameter λ is given in the interval $\left[10^{-6}, 10^{-3}\right]$.

The objective function that maximizes the total value of the prioritized orders is transformed into a constraint:

$$\sum_{o\in O^{\varepsilon x}} K_o\left(\sum_{a\in A}\sum_{t=1}^{T} y_{a,t,o}^{A} + \sum_{t=1}^{T+F} y_{t,o}^{DC}\right) + \sum_{o\in O^{IN}} K_o \sum_{t=1}^{T} y_{t,o}^{IN} - S_2 = \varepsilon_2, \tag{42}$$

where ε_k is calculated as follows:

$$\varepsilon_k = f_k^{MIN} + t\left(\frac{r_k}{q_k}\right),$$

where f_k^{MIN} is the minimum that is obtained from the payoff table, and t is the counter for the specific objective function. The number of intervals for objective function k^{th}, q_k, influences the potential number of different Pareto-optimal solutions that can be found.

4. COMPUTATIONAL STUDY

This section reports on our computational experiments using the mathematical model that was provided in Section 3. The model was implemented using AMPL and solved using CPLEX (Version 20.1.0.0 – 64-bit). The tests were run on a computer with 16 GB RAM and a 3.7 GHz Intel Core i5-9600K CPU with six cores.

4.1. Test instances

Based on the operations of our focal company, we created ten different test instances. The instances varied in terms of the numbers of farms, plants, species, size classes, qualities, certifications, diseases, and orders; the values for each of these elements are presented in Table 1. While the number of plants, species, size classes, qualities, certifications, and diseases are based on real-world data, the number of orders, the demand and supply, and the order priorities were generated to span a range of potential realistic situations.

Table 1. *Test instances*

Entity	**Instance**				
	1A, 1B	2A, 2B	3A, 3B	4A, 4B	5A, 5B
Farm	2	3	4	5	6
Plant	2	3	4	5	6
Species	2	2	2	2	3
Size class	3	4	4	6	6
Quality	2	3	4	4	4
Diseases	2	3	4	4	5
Certifications	2	3	4	4	5
Orders	50	100	150	200	250

In the instances, the orders with low demand had a high priority, and the orders with high demand had a low priority. The prioritization coefficients had values from a discrete interval of integer numbers from 1 to 10 (where 10 was the highest). The orders were divided into groups of ten orders, each requesting the same combination of fish but a different number of boxes. The demand for each combination in an order was set to be a random number between 100 and 1000 boxes.

The required days of delivery were set so it was possible to deliver all of the orders on any day throughout the planning horizon. The requirements about diseases and certifications were set so that all of the orders accepted all fish (independently from its health status and certifications). The only source of incoming supply was set to be farms. The initial inventories were set to zero for all of the plants and the distribution center. The inventory capacity was set to zero for all of the plants, so any supply that needed to be stored was required to be sent to the distribution center.

All of the orders allowed for a deviation of 10% from the number of boxes that were requested for a given combination; this allowed for flexibility in demand. All of the orders were set to be external orders. The weights for the delivery methods for the external orders were set so that direct delivery was weighted higher than delivery through the distribution center. Parameter T was set to 5, the lead time from a plant to the distribution center was set to 1 or 2 days, and the longest lead time F was 2.

The ten instances were divided into two sets on the basis of the balance between the supply and demand for a given combination of fish. In the first set (A), the five instances had more supply than demand, and in the second set (B), the five instances had less supply than demand. The instances in the two sets were identical except for their available supplies. The size of each instance is presented in Table 2.

Table 2. *Problem size*

Instance	Variables	Constraints
1A, 1B	30,562	28,934
2A, 2B	265,504	138,142
3A, 3B	935,378	337,942
4A, 4B	2,909,200	744,230
5A, 5B	7,834,500	1,608,570

4.2. Results

This section presents an analysis of the results for each problem instance as was found by using the proposed solution method. We started by providing pay-off tables for the instances in Tables 3 and 4. The payoff tables were calculated as described by Chowdhury and Tan (2004), meaning that we first optimized each individual objective separately as a primary objective and then optimized for each remaining objective subject to fixing the value of the current primary objective to its optimal value using a hard constraint.

Table 3. *More supply than demand*

Instance	Max		Value	Seconds
1A	W_1	W_1	27,706	0.1
		W_2	275	0.1
	W_2	W_1	27,706	0.1
		W_2	275	0.1
2A	W_1	W_1	56,665	0.1
		W_2	550	0.3
	W_2	W_1	56,665	0.1
		W_2	550	0.0
3A	W_1	W_1	82,728	0.1
		W_2	825	0.4
	W_2	W_1	82,728	0.1
		W_2	825	0.0
4A	W_1	W_1	110,210	0.2
		W_2	1100	0.6
	W_2	W_1	110,210	0.3
		W_2	1100	0.1
5A	W_1	W_1	141,377	0.2
		W_2	1375	0.9
	W_2	W_1	141,377	0.3
		W_2	1375	0.1

Table 4. *Less supply than demand*

Instance	**Max**		**Value**	**Seconds**
1B	W_1	W_1	19,398	0.1
		W_2	252	2.9
	W_2	W_1	18,756	267.7
		W_2	260	0.2
2B	W_1	W_1	39,673	1.2
		W_2	506	3.4
	W_2	W_1	38,301	31.9
		W_2	520	0.1
3B	W_1	W_1	57,919	0.4
		W_2	763	6.1
	W_2	W_1	56,186	1,551.3
		W_2	782	0.2
4B	W_1	W_1	77,160	0.7
		W_2	1021	11.3
	W_2	W_1	74,823	8,344.4
		W_2	1044	0.8
5B	W_1	W_1	98,978	0.6
		W_2	1275	7.0
	W_2	W_1	96 385	1,727.9
		W_2	1304	0.4

If a solution that was found in an instance was the same regardless of the order in which the objectives were solved, then only one Pareto-optimal solution existed for that given instance. In those instances with more supply than demand, the solutions within each instance were the same for both objectives regardless of the order in which they were solved. Both objectives had different values to maximize, but their overall intention was to serve all orders. Having more supply than demand enabled the possibility of fulfilling all orders and caused no conflict between the objectives.

The opposite could be observed for those instances with less supply than demand: the solutions were affected by the order in which the objectives were solved. The improvement of one objective led to a worsening of the other objective, which was indicative of the existence of more than one Pareto-optimal solution. Having less available supply than demand prompted the need to decide which order was most important to fulfill. The two objectives differed in how they treated orders; one objective considered the number of boxes that were specified in an order to be an indicator of importance, while the other objective focused on the value of the priority coefficient.

The time that was required to solve each instance was relatively low in those cases with more supply than demand. When solving those instances with less supply than demand, however, the solution time increased. This was especially evident when maximizing the total number of boxes that were delivered to all orders (W_1), while the optimal total value of the fulfilled prioritized orders (W_2) was used as a constraint.

Having observed that each of the instances with less supply than demand had at least two Pareto-optimal solutions, a further analysis was conducted for this case. We found that the same number of orders was fulfilled regardless of the order in which the objectives were handled; the only difference was in the individual orders that were fulfilled. In general, when maximizing the number of boxes that were delivered first, more low-priority orders were fulfilled as compared to when maximizing the total values of the prioritized orders first. This was due to the fact that those orders with low prioritization coefficients requested the greatest number of boxes for a given combination of fish attributes. Also observed was the pattern that the highly prioritized orders for a given combination were always fulfilled (independently from the order in which the objectives were solved). This was due to the fact that higher-priority orders requested relatively low numbers of boxes as compared to the available supply, making it possible to fulfill many orders with high priorities.

To further investigate the conflict between the two objective functions for those instances with less supply than demand, we next used the augmented ε-constraint method to generate the Pareto fronts for the five instances with this characteristic. This meant that the model was solved multiple times for each instance in order to generate different Pareto-optimal solutions. In each run, we used a maximum time limit of 700 seconds and specified an acceptable optimality gap. The acceptable optimality gap was set slightly higher (for instance, 5B, as this instance is larger and more difficult to solve). Table 5 summarizes the settings that were used to solve each instance.

Table 5. *Optimality gap and grid points*

Instance	Optimality gap [%]	Grid points
1B	0.01	9
2B	0.01	15
3B	0.01	20
4B	0.01	24
5B	0.05	30

Having instances of increasing problem sizes opened up observations into whether any iteration was unable to meet the required optimality gap before its time limit was reached. Table 6 gives a summary of the number of unique solutions for each instance, the maximum and minimum optimality gaps that were reached, and the number of times that the time limit was reached. Since the number of found unique solutions was identical to the number of grid points that were used, the resulting approximate Pareto fronts indicated that there may have been many more Pareto-optimal points that were not found.

Table 6. *Summary of instance solutions*

Instance	Unique solutions	Max gap [%]	Min gap [%]	Number of times time limit reached
1B	9	0.01	< 0.001	0
2B	15	0.01	0	0
3B	20	0.017	0	1
4B	24	1.265	< 0.001	18
5B	30	1.161	0.033	18

When creating the Pareto fronts for Instances 1B and 2B, all of the iterations found Pareto-optimal solutions with optimality gaps of 0.01% or lower within the time limit. As the size of the problem increased, fewer iterations could find a solution that satisfied the optimality gap that was required within the time limit. This was especially apparent for Instances 4B and 5B – both of which having 18 runs in which the required optimality gap was not met within the time limit. At most, two of the solutions had optimality gaps of more than 1%. This caused an issue regarding the certainty of having actually found a Pareto-optimal solution since the gap to optimality was so large. The estimated Pareto fronts for Instances 2B and 5B are illustrated in Figure 4. For the other instances, the Pareto fronts had similar shapes but with varying numbers of different points (as indicated in Table 6).

a) b)

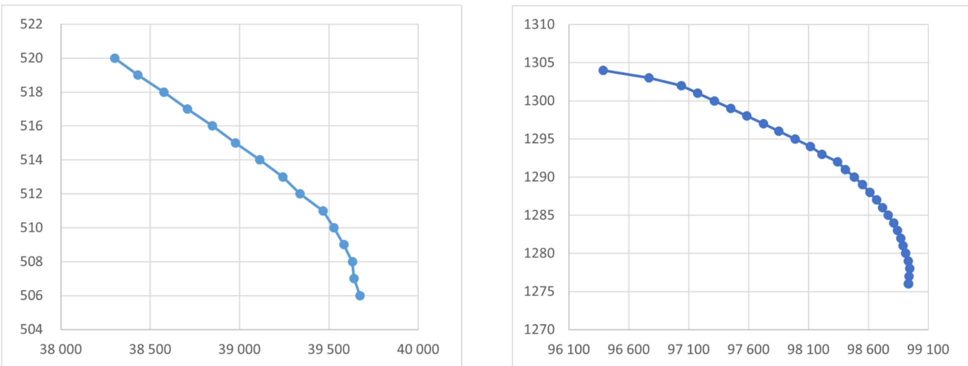

Fig. 4. *Pareto fronts for two individual instances, with W_1 on first axis and W_2 on second axis: a) Instance 2B; b) Instance 5B*

5. CONCLUDING REMARKS

This paper has presented a conceptual model of the allocation process that takes place in a company in the fish-farming industry. The company allocates freshly harvested fish to customer orders in order to satisfy the demand of its customers. A bi-objective mathematical programming model was formulated to capture the essence of the planning problem, aiming to provide decision support for the planners at the company.

The two objectives that were modeled were as follows:

1) maximize total number of boxes of fish delivered to all orders,
2) maximize total value of fulfilled prioritized orders.

These two objective functions represent a realistic allocation dilemma. When reviewing the existing literature, no previous multi-objective assignment model was found that considered any similar allocation problem. To solve the instances of the mathematical model, the augmented ε-constraint method (AUGMECON) was used, as it allowed us to find Pareto-optimal solutions in an efficient manner.

A computational study was performed using ten different instances. The study demonstrated an example where those orders that requested high numbers of boxes were set to have low priorities, while those orders that requested low numbers of boxes had high priorities. The results showed that, in a scenario with more supply than demand, no conflict could be observed between the two objectives of the model. This showed that, in cases where there is more supply than demand, a single-objective formulation may be sufficient.

When there was less supply than demand, conflicts occurred; these resulted in different Pareto-optimal solutions depending on which objective was solved first. In these cases, using multi-objective optimization was appropriate. The computational effort that was required to solve the instances also increased significantly when there was less supply than demand. Approximate Pareto fronts were created for all instances with lower supply than demand. The results showed that, as the size of the problem grew, fewer solutions with optimality gaps that satisfyed the requirement were found before the time limits were reached.

Solving a problem instance that is described with the model provides a plan for the allocation of fish to customer orders for a given horizon. This plan can be used to help planners determine how their supplies should be allocated to their orders; this may help their companies make better trade-offs in situations where planning is difficult (such as when the supply is insufficient to cover the demands from all of the orders).

Additional objectives could have been included in our study. As an example, a focal company may wish to maximize its profits in addition to the amount of fish that is delivered or the number of prioritized orders that are fulfilled; this includes selling their fish to the spot market for a better price. A limitation of our study was the lack of economic data on the transactions. For future research, we also propose to investigate the effect of uncertainty on supply forecasts. The quality and size of harvested fish are only known probabilistically until the fish is slaughtered this uncertainty may influence the quality of the planned order allocations.

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