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PROFESSOR ANDRZEJ LASOTA – A BIOGRAPHICAL NOTE

Andrzej Lasota was born in Warsaw on January 11th 1932. His otherwise tranquil childhood was disturbed by the outbreak of World War II. His father, a career's officer of the Polish Army, was actively involved in the Polish-German war of September 1939. Warsaw having surrendered, the Lasotas were evacuated to the East Borderland, which – following the Soviet invasion – fell under the Red Army's occupation. Owing to the extraordinary intuition and swiftly made decisions, the Lasotas were repeatedly spared being arrested and the family father was saved from the fate of Polish officers murdered in Katyń. Life under the Soviet occupation, the feeling of constant danger and the resulting numerous changes of residence – they all etched irreversibly on the psyche of the would-be mathematician. They did also affect his philosophy of life. After recurring vicissitudes, the Lasotas managed tom return to Warsaw, where they stayed until the Warsaw Uprising of 1944. They were luckily enough to again save their lives in that turmoil. Warsaw having been destroyed, the Lasotas took refuge in Kraków, from where they later moved to Poznań. There, in 1951, Andrzej Lasota passed his maturity examination at Państwowa Szkoła Ogólnokształcąca Stopnia Licealnego [*State Secondary School*].

In the same year, following his interests, Andrzej Lasota enrolled at the Jagiellonian University of Kraków to study physics. Chance so ordained it that that was Professor Tadeusz Ważewski, a mathematician of a profound insight and also a very talented teacher, who thought mathematics to physics majors then. The young student diligently attended the classes and fervent discussions with the renowned professor brought about a rather unexpected decision. Let us hear as Professor Lasota recalled it itself ([5, 6]): "*While I started to study physics, because it interested me, enchanted with Ważewski's lectures, I changed my major to mathematics. Ważewski was meticulously precise in his mathematical reasoning and precise to the extreme, but he never got lost in details. He could see the genuine beauty of mathematics: its physical and natural background. He was uniquely able to show how mathematics is created.*" It was also of major importance for Andrzej Lasota's decision that if it was a student who was right (as it was the case with student Lasota), Professor Ważewski did not hesitate to admit it publicly. Not only did this openness to mathematical reasoning attracted Andrzej Lasota to the mathematical community, but it also per-

vaded his entire later activity in the field of mathematics, which invariable attracted to him so numerous co-workers representing so varied fields of science and technology.

In 1955, Andrzej Lasota submitted his M.Sc. thesis prepared under the guidance of Professor Ważewski, then heading the Chair of Differential Equations at the Jagiellonian University. Professor Ważewski employed the new graduate at his Chair. In 1960, at the mathematical Institute of the Polish Academy of Sciences, Andrzej Lasota was conferred upon the Ph.D. degree, based on the thesis *On Certain Limit Problem for the Oscillating String Equation*. Quite naturally, the thesis was prepared under the tutorial of Professor Ważewski. Four years later, at the Faculty of Mathematics, Physics and Chemistry of the Jagiellonian University, Doctor Lasota submitted his thesis *On the Existence and Uniqueness of Solutions to Nonlinear Differential and Integral Equations*, which earned him the degree of doctor habilitated.

Andrzej Lasota was still a university student, when he recognised the Jagiellonian University his *Alma Mater* and remained faithful to this allegiance for ever after. However, in his later years, he also developed close bonds with the Silesian University.

It is worth noting here that Professor Lasota never lost his interest in physics and he saw mathematics as an important component of reality. May he speak of it for himself ([1, 5]): "*I believe mathematics to be the very structure of the world. Not a description of the structure, but the structure itself. There is no doubt that a mathematician can create strange objects and he may think that he has strayed far from reality. But this is a resemblance of truth only. If his creation is good mathematics, it will sooner or later prove to be a component of reality. If it is bad mathematics, it will be no more, but a cluster of shards of the real world, in the same manner in which a dream is a cluster of our everyday experiences. A dream may be bizarre, but even in a dream no one can speak a language he does not know.*" Professor Lasota was untiring in his pursuit of the mathematical structure hidden in the world around us. In his endeavours, he cooperated with mathematicians, physicists, physicians, biologists and engineers. Since the beginning of his research activity in mathematics, he was involved in these branches thereof which reflected reality in the best way possible. In the search for reality, Professor Lasota ventured into various domains of the mathematical realm, in each of them leaving his indelible trace in the form of outstanding results.

His contributions to the theory of differential equations include research into the interrelation between the existence and uniqueness of solutions to linear (both ordinary and partial) differential equations and linear integral equations. He also blazed trails for using multifunctions in proving the existence of solutions to boundary value problems, found a relation between the existence and uniqueness of solutions to periodic differential equations; proved an implicit function theorem for non-differentiable functions; applied this theorem to proving the existence of a solution to a boundary value problem and of a periodic solution to an ordinary differential equation; and demonstrated that, in a Banach space, the existence of a solution to an ordinary differential equation with a continuous right-hand side is a generic property. In this scope, his prominent co-authors include Professors Zdzisław Opial, James A. Yorke, Czesław Olech and S. N. Chow. In the theory of nonlinear operators, Professor Lasota proved an analogue of the Fredholm Alternative using the multifunction technique.

In measure theory, in cooperation with James A. Yorke, he solved Ulam's conjecture concerning the existence of an invariant measure for a piecewise monotone transformation. He suggested new ways of defining a measure dimension and laid foundations for chaos theory for partial differential equations. In probability theory, Professor Lasota discovered the lower bound function method, which proved a powerful tool for studying the stability of Markov operators acting on densities. In fractal theory, he proposed to separate a new class of sets, which have subsequently been called semifractals. This notion connects fractal theory with the theory of Markov operators acting on measures and – in a more general setting – the fixed point theory for multifunctions with the problem of invariant measure for Markov operators. Applying mathematics to problems in biology and medicine, in cooperation with Dr Maria Ważewska-Czyżewska, Professor Lasota built a model describing the blood cell reproduction process. In cooperation with engineers, he used ergodic theory to design drilling tools and fractal theory to describe the grinding process.

In 1972, Andrzej Lasota became professor extraordinary[1)]. In 1972–1976, he headed the Chair of Probability Theory and in 1972–1975, served as the Dean of the Faculty of Mathematics, Physics and Chemistry of the Jagiellonian University. Searching to improve his living conditions (those were the years of a deep collapse on the residential construction market in Poland, and especially in Kraków), Professor Lasota joint the Silesian University of Katowice in 1976. However, he never severed his bonds with the Jagiellonian University, where he for decades continued to host a seminar and give monograph lectures. But with each year of his work in Katowice, Professor Lasota was developing his emotional bonds with the Silesian University, where he headed first the Chair of Biomathematics and then the Chair of Probability Theory.

In 1979, Andrzej Lasota became professor ordinary[2)]. He was a Corresponding Member of the Polish Academy of Sciences since 1983 and a Full Member since 1994. He became a Corresponding Member of Polska Akademia Umiejętności [*Polish Academy of Arts and Sciences*] of Kraków in 1997 and a Full Member in 2001.

In 1986–1988, Professor Lasota worked for the Maria Curie-Skłodowska University of Lublin, from where he then returned to Katowice, to work for the Silesian University. During his Lublin years, Andrzej Lasota faced a number of health and family problems. Consequently, Lublin is the only venue where he worked but did not establish his own research school.

Professor Lasota cooperated with various research centres across Poland. He often promoted new trends in both scientific research and mathematical education. Together with Professor Zdzisław Opial, he introduced numerical methods as both a research and curriculum subject in the Kraków academic centre and in 1967–1968 served the head of a workshop at Instytut Maszyn Matematycznych [*Institute of Computers*] of Kraków. Later, as the Dean of the Faculty of Mathematics, Physics and Chemistry of the Jagiellonian University, he made a key contribution to opening

[1)] *Professor extraordinary* and *professor ordinary* are academic titles granted to scholars by the President of the Republic of Poland (until 1989, by the Chairman of the State Council) upon a motion submitted by a higher education or other scientific institution.

[2)] See footnote 1 above.

computer science as a major at the University. Since 1956, he invariably cooperated with the Mathematical Institute of the Polish Academy of Sciences, first as a research assistant, than assistant professor [*adiunkt*] and finally professor. Long and extremely fruitful was his cooperation with the AGH University of Science Technology of Kraków, especially with the Institute of the Foundations of Machine Design and Chair of Mechanical Technology. This cooperation led to not only a series of interesting papers and doctoral theses (including by Professor Piotr Rusek, Doctor J. Kołodziej and Professor M. Bałaziński), but also a valuable patent. Certain results in this scope are discussed in Piotr Rusek's paper included in this volume. It was in the course of this cooperation that Professor Lasota's interest and expertise in physics proved extremely useful. It was quite often that engineers cooperating with Andrzej Lasota were surprised to see that his insight in technology problems was deeper than theirs. It should also be emphasised here that while Professor Lasota played no formal role in the above mentioned doctoral procedures, he – as the doctors attest themselves – assisted them in developing mathematical models of the phenomena studied and often inspired further technological experiments. Moreover, he also set off to develop a mathematical model of friction theory, but – for his numerous and various interests – he did not manage to complete the project.

Among his results in applied mathematics, Professor Lasota valued most those achieved in cooperation with Professor Maria Ważewska-Czyżewska, concerning a mathematical model of blood cell reproduction. Professor Ważewska applied the results achieved in her clinical practice, in the treatment of certain types of post-treatment anaemia. Let us read how professor Lasota described this cooperation himself ([2]): "*Professor Maria Ważewska had a consummate knowledge of haematology and was aware that certain mechanisms analysed in haematology admit mathematical modelling. She once shared this idea with me; after a time I suggested several mathematical models and from the suggested ideas she selected what she seemed most promising from the biological point of view. We finally built a model which is called a nonlinear differential equation with retarded argument in the mathematical jargon. Certain properties of this model evade or even seemingly defy common sense, while well corresponding to the phenomenon being described. This is one of those instances to which a friend of mine, Professor Opial referred saying that mathematics has proved wiser than the mathematician. Using solutions to the differential equations we had been studying in the projected therapy, Professor Ważewska materially improved the health of several post-treatment anaemia patients. If my research contributed to it, even to a minor extent, this may be the most valuable of my life's achievements.*"

The untimely death of Professor Ważewska-Czyżewska put an end to this cooperation, but the ideas initiated in its course have been developed in a number of scientific centres. Professor Lasota himself continued research in this field: for a number of years he cooperated with the renowned biologist, Professor Michael C. Mackey of the McGill University of Montreal, Canada. The main subject of their joint research was models of cell growth and differentiation. Professor Mackey described the cooperation in detail in his essay *Adventures in Poland: Having Fun and Doing Research with Andrzej Lasota* published in *Matematyka Stosowana* in 2007. For one of the equations describing the development of blood cells, Professor Lasota coined

the name of the Ważewska-Czyżewska equation and this name has since established itself in the literature all over the world. A number of papers have been devoted to mathematical properties of the equation and the model leading to the equation has proved applicable in other branches of science.

Professor Lasota visited numerous research centres and maintained far-reaching international contacts. His longer stays abroad include: the semi-annual post-doctoral placement at the Moscow University in the Soviet Union (currently: Russia), annual stay at the Institute of Fluid Dynamics and Applied Mathematics of the University of Maryland, College Park, MD (USA) in 1969–1970 and annual stay at the Institute of Mathematics of the Michigan State University, East Lansing, MI (USA) in 1982–1983. Moreover, as a visiting professor, he went to numerous other institutions, including the University of Florence, Italy, the McGill University of Montreal, Canada, the University of Oxford, UK, the SISSA of Trieste, Italy, the University of Udine, Italy, the University of Siena, Italy, and the University of Rome, Italy. Especially frequently did Professor Lasota visit the University of L'Aquila, Italy, where he went ten times as a visiting professor. Occasionally, he went there also during his visits to other Italian research centres. Sometimes, he used the opportunity to 'escape' for skiing in Gran Sasso. Andrzej Lasota established long-term cooperation with a number of scientists, including Professors James Yorke, Michael Mackey, F.S. De Blasi, G. Pianigiani and Józef Myjak, representing such institutions as the University of Maryland, the University of Michigan, the University of Florence and the University of L'Aquila. This cooperation is reflected in numerous research papers and two monographs by A. Lasota and M. Mackey. The first of them, entitled *Probabilistic Properties of Deterministic Systems*, was published by Cambridge University Press in 1985. The second one, being an enriched edition of the former, was published in 1994 by Springer-Verlag as *Chaos, Fractals and Noise: Stochastic Aspects of Dynamics*. According to *The Mathematical Reviews*, "*... the authors made an excellent job in bringing to the attention of a broad audience how nonlinear dynamical systems may be studied with use of linear operator theory, and probability theory.*" James Yorke describes Professor Lasota's contribution to mathematics as follows ([3]): "*Mathematics describes the world mainly with use of two of its branches: the theory of dynamical systems and differential equations, as well as probability theory and statistics. Nonlinear dynamics is a theory describing changes in time: changes in objects, appliances and whole systems. It takes the central place in each domain on knowledge. While creating pure mathematics lying at the border between these two branches, Professor Andrzej Lasota teaches us how to look at the world. The probabilistic treatment of dynamical systems that he has proposed is, no doubt about that, a revolution. The Berkeley school of dynamical systems adopted such a method, which has become one of the pillars of this theory.*"

Professor Lasota was repeatedly invited to international mathematical events, as well as to give plenary lectures. While with passing years he grew more reluctant to travel, the number of conferences he attended is still considerable. Let us recall just these few: the IMU Congress Warsaw 1983 (with a plenary lecture), the Władysław Orlicz Lecture at Collegium Mathematicum in 2000 and the Wacław Sierpiński Lecure in 2002 (both at the Adam Mickiewicz University of Poznań).

Professor Lasota conferred the Ph.D. degree in mathematics upon 21 young mathematicians. The majority of them are already doctors habilitated, with six holding the official title of professor. Also numerous foreign mathematicians consider Professor Lasota their teacher and master, among them, certainly, Professors F. D. De Blasi and G. Pianigiani (both Italian), S. N. Chow and R. F. Bernfeld (both American), as well as J. Komornik (from the Czech Republic).

Andrzej Lasota was an extraordinarily generous man. While usually superior to his co-workers in terms of knowledge and ingenuity, Professor Lasota never made them feel it, but always appreciated their contribution, however insignificant it might have been. It was a great pleasure to work with him. Always an inexhaustible source of excellent ideas, he gave all his attention to his interlocutor's suggestions, which he often summed up saying "*It's interesting, even very interesting; what a pity it's false*", and presented a counterexample to support his statement. He took the obstacles he faced as a challenge to his will and fought back; he practically never gave in. He bestowed his own ideas on not one of his students. Then he consistently deemed the ideas which he shared with others as their property, and not his own. I remember that one day I expressed my doubts as to the authorship of some result credited to one of his disciples. When we were using the result, I asked him how he had got the idea. Taken aback, he explained it and went on with the proof, but made sure that the result referred to should be credited to its official author. I felt he was hurt by my having made him divulge the information.

Andrzej Lasota was for years active in various sports. As a young man, he won the Małopolska Championship in wrestling. For a long time he practised mountain-climbing and recorded a number of difficult and extremely difficult routes in the Tatras. He also regularly practised down-skiing, even after retirement. He was a tough man, as in sports, so in his work. Once he fell on a ski slope in Szczyrk and broke his collarbone. Skiing conditions being good though, he would not give up and continued to ski with his broken arm tied up to his torso, until ski-lifts closed. No surprise that the emergency would not come for such fit a patient and it was with much trouble that his companions brought him to the hospital in Bielsko-Biała by public transport. Andrzej Lasota fought proofs and theorems in a similar manner. Difficulties spurred him to try harder and new horizons attracted him.

Professor Lasota was a highly respected authority in the scholarly community, not only among mathematicians. The range of his interests was extraordinarily broad. In addition to mathematics and physics, he was also interested in medicine and biology. He believed there is one mathematics only and that it is a component of reality. In particular, he would not acknowledge the division into pure and applied mathematics. He himself commented on it as follows: "*My view on mathematics is somewhat different from that of the majority of my colleagues. Perhaps the reason is I have come to mathematics from outside. I was interested in biology and economics, I enrolled at the university to study physics and only later did I become a mathematician.*" A special place in his interests was given to philosophy. For a time, he even lectured philosophy. It is worth remembering here that his lecture *Mathematics and Philosophy* given at the Kraków Philosophical Symposium in 1995 evoked quite a response among both the followers and opponents of his ideas (cf. Professor Roman Duda's, Professor

Michał Heller's and Professor Jacek Urbaniec's statements in *Otwarta nauka i jej zwolennicy* [*Open Science and Its Followers*], OBI, Kraków 1996). It is interesting that it were philosophers who most agreed with Professor Lasota's view. In the paper devoted to Lasota's views, Rev. Professor Michał Heller approved them to a major extent. There is no doubt that Professor Lasota was a classical scholar in the full sense of the word.

Professor Lasota's ability to accurately formulate problems in economics is attested to by the following event. As a young teaching assistant, he met a female student of economics, who complained that she had not been able to understand a lecture on political economics. To the next date, instead of a bunch of flowers, the young mathematician brought a handwritten handbook of political economics which he wrote in the meantime. The student passed the examination with A+, married the author of the handbook and the duplicated manuscript for years since helped economics students to understand political economics.

I think that the grim experiences of the childhood made Andrzej Lasota detest totalitarian systems. For the same reason he was reluctant to take up any political activity, whether in support of or against the government. He strongly dissuaded his students from political or social activity, which he deemed a waste of time. And yet he was ever a keen observer of the political scene. As he was leftist by his beliefs, he unrelentingly criticised the activities of leftist parties. On the other hand, he was completely indifferent to rightist views; consequently, he did not bother to comment on the activities of right-wing parties. I believe that also his critical views on certain aspects of the Church's activity were founded in his social beliefs, while sometimes they were erroneously thought to be a manifestation of his alleged atheistic beliefs. Andrzej Lasota was always a man in the search for the truth. He respected everyone's beliefs, whatever they might be. He disliked, though, duplicity and hypocrisy. Having sensed an opportunistic background in his interlocutor's views, he became sharp-tongued and ruthless. But he delighted in conversations with people of well-established beliefs. For instance, he for hours conversed with Professor Andrzej Pliś, a man of deeply rooted Christian faith. Andrzej Lasota repeatedly recalled these discussions after Professor Pliś had died and missed them.

Professor Lasota was on numerous occasions awarded for his achievements. The awards he was granted include: the Stanisław Zaremba Award of the Polish Mathematical Society (PTM) in 1967, the Scientific Award of Division III of the Polish Academy of Sciences (PAN) in 1969, the PTM's Award for Achievements in Applied Mathematics in 1974, the Award of the PAN Scientific Secretary in 1974, a collective award (with Maria Ważewska-Czyżewska) of the Medical Sciences Division of the PAN in 1997, the Minister of National Education's award for the book *Chaos, Fractals and Noise* in 1994 and the President of the Council of Ministers' award for outstanding academic achievements in 2004. He was decorated with the Cavalier's Cross of *Polonia Restituta* in 1974, the Medal of Komisja Edukacji Narodowej in 1976 and the Officer's Cross of *Polonia Restituta* in 1988. In recognition of his outstanding achievements, in 2001, the Senate of the Silesian University granted Professor Lasota the title of *doctor honoris causa* and, in 2003, the General Meeting of the Polish Mathematical Society named him its Member of Honour.

It appears appropriate to conclude this brief note with Professor Lasota's own opinion, the opinion which I find hard not to share ([1]).

"*We are currently witnessing a rapid, intensive development of mathematics. More mathematicians lived in the 20th century than in all earlier centuries combined. After World War II, millions of new theorems have been proved and published. It is, though, only an appearance, camouflage. Nobody is able to use millions of theorems. At the beginning of the previous century, mathematicians succeeded in axiomatising probability theory; laying foundations for topology, functional analysis and several other branches forming the basis of modern mathematics. Since then, all mathematicians have been playing with the same building blocks: sets, relations and functions. The rate of expansion of the mathematical methods, which in past centuries conquered astronomy, physics and chemistry, decreased. Attempts at applying mathematics to model social phenomena have been a shame, and in the case of biological systems they have proved mathematicians unequal to the task. Why is mathematics broadening its reach, but not going up? I think there are two reasons: one psychological and the other one sociological. First, modern mathematics is already beautiful, precise and flexible. In short, it is good. And striving to well oft we mar what is better. We do not seek anything new, because it is easier to develop old, beautiful and still fruitful theories. Second, in olden times a scholar was a kind of madman. Nowadays, being a scholar is a stable and quiet job. Powerful institutions have been established to see to it that appropriately qualified individuals only become scholars. Career paths and procedures of raising funds for research projects have been codified. To know how to fill forms is a necessary condition for promotion and grant winning. Rejecting unfit eccentrics includes rejecting a lot of geniuses. The implementation of breakthrough projects has been frustrated. The academic bureaucracy, consuming a portion of funds earmarked for research projects, is active in one field only: issuing nonsensical regulations and directives. However, the progress may be decelerated, but not halted. People will be born who will change the image of contemporary science. In several decades from now we will operate new notions and new effective methods we do not even dream of today. The quantity will not transform into quality. Quite contrary, a new quality will emerge in opposition to quantity. Such mathematics is searched for, when we dream of the universal potential of catastrophe theory, fuzzy sets or fractal geometry. So far, they have only been attempts raising enormous controversies. Let it be enough to recall the confusion brought about by catastrophe theory. Similar controversies, in a smaller scale, though, have stemmed around fuzzy set theory. Being a confirmed optimist, I set my high hopes on fractal theory, as it has enabled us to see that most intricate processes admit modelling as dynamical systems in a space of measures. Both relatively simple substances and complex biological systems appear to be fractals. Probability theory needs to be appreciated more. Mathematicians, as a rule, neither like nor understand probability theory. In Bourbaki's survey of achievements of contemporary mathematics, probability has not even earned a mention. Yet it is in this field that most spectacular achievements have been recorded. In terms of theoretical background, probability theory ranks among most advanced branches of mathematics. In terms of applications, it has left the other branches far behind. Very soon the old*

adage that probability theory is one of the major branches of natural philosophy with the rest of mathematics being its subsidiary science will no longer be a joke."

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