AGH Journal of Mining and Geoengineering • vol. 36 • No. 3 • 2012

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RESEARCH ON E.C. BINGHAM CONSTITUTIVE MODEL OF ROCK UNDER IMPACT LOAD FOR DEEP WELL[***]

1. Introduction

In underground mining, the rock mass are often affected by the impacts of dynamic loads, such as the impacts from tunneling by blasting, roof and floor breaking and so on, leading to rock and gas bursts [1] due to the local instability of mining area and surrounding rock. To study the mechanisms for all the effects these impacts exercise on the rock mass and the possible measures we can take to resist blasting impacts, it is necessary to master the dynamic performances of the rock and rock mass and to understand the constitutive relations thoroughly, with focus laid on the latter, because the constitutive relations are necessary parameters for the study of the laws governing the spreading of stress waves and blasting resistant designing [2].

In studying constitutive relations for underground soft rock, the constitutive relation model serves as the key, for the precision of the model will to a large extent determine how the calculation results for the impacts on undergrou nd soft rocks reflect their dynamic mechanic behaviours [3]. At present, the most influential dynamic constitutive model for rocks is the overstress model [4] devised by U.S. Lindholm in 1974. However, the physical parameters

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[***] The work is supported by New Century Excellent Talents of Ministry of Education project NCET-08-0819, Huo Yingdong Education Fund 121050, the China National Science Foundation of China 51074005/51134012,Key Projects on College Natural Science Research of Anhui Province KJ2011A085, Academic research activities subsidies for Academic and technical leaders and Backup candidate in Anhui province, the Key project of science and technology research of the Ministry of Education 210096,and the Science and Technology Projects 2009A05014 in Huainan City.

involved in the model are not precisely defined. In the 80s of the last century, Kinosila Sigenori, Satouk Azuhiko and Kawakita Minoru revised the overstress model [5], and Yu Yalun from China devised his revised overstress model [6] by revising the above-mentioned models. However, the revised model cannot reflect how modulus of elasticity varies with the changing rate of load. To overcome the defects, based on the rheological model put forward by Boeding, in 1996 Zheng Yonglai and Xia Songyou devised their continuously damaging viscoelastic constitutive model [7]. The model is characterized by its damaging feature. In other words, in the model, viscous elements are also regarded as damaging though they are actually not. Therefore, the model is fuzzy for its physical concepts. In last few years, Shan Renliang has conducted investigations into the constitutive features of marble and granite and established the impact-aging damaging model based on the characteristics of the stress-strain curve of the constitutive features. However, up to the present, not a single constitutive model has been established specially for underground soft rocks.

By focusing the study on such typical soft rocks as mudstone and sandy mudstone, the present author has conducted investigations into the dynamic compression performance. And based on the dynamically mechanic properties and by making improvements on existing constitutive models, the present author has introduced a viscoelastic statistic model for damaging which is appropriately applicable to the study of underground soft rocks.

2. The Underground Soft Rock SHPB Experiment

2.1. The SHPB Experiment

Experiments were conducted with typical soft rocks such as the mudstone and the sandy mudstone buried over 1000 m deep in Huainan and Huaibei Areas of Anhui Province, aiming to have a clear understanding of the dynamic compression performance. The static uniaxial compressive strengths are 14.5 and 21 MPa respectively. The SHPB experiments were conducted to study the uniaxial compression performance and the devices used for the experiment are shown in Figure 1.

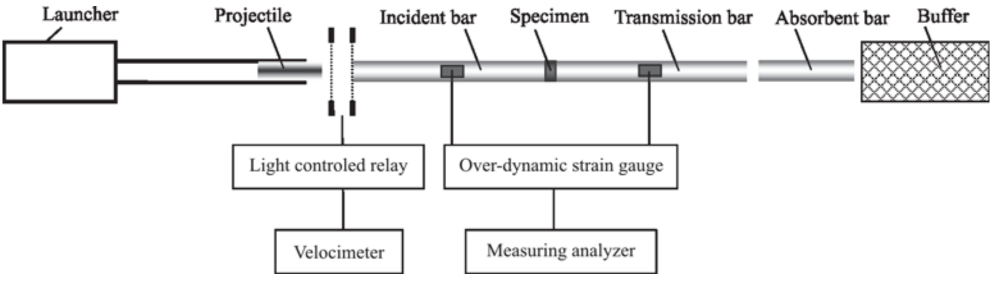

Fig. 1. SHPB set-up

The diameter of the SHPB bar is Φ 14.5 mm wide, the incident bar is 500 mm long, the transmission bar is 500 mm long, the modulus of elasticity for the bar is 210 GPa, the Poisson's ratio is 0.30, the density is 7850 kg/m^3 and the projectile is 200 mm long. 2.2 Analysis of the Dynamic Stress-Strain Curve

At different impact velocities, the typical stress-strain curves for mudstone and sandy mudstone are shown in Figure 2 and 3.

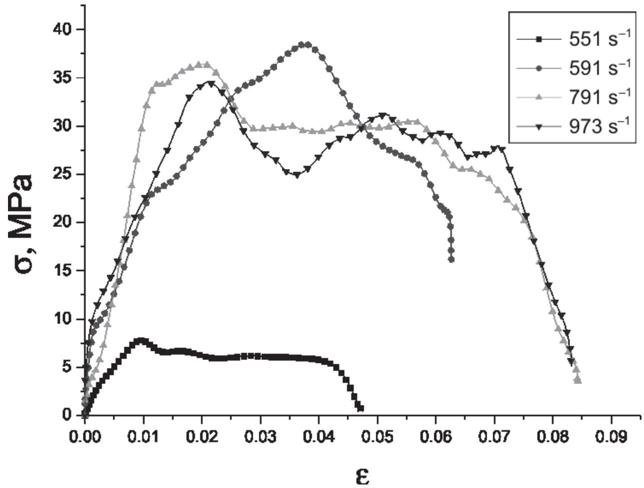

Fig. 2. Stress-strain curves for Mudstone in different strain rates

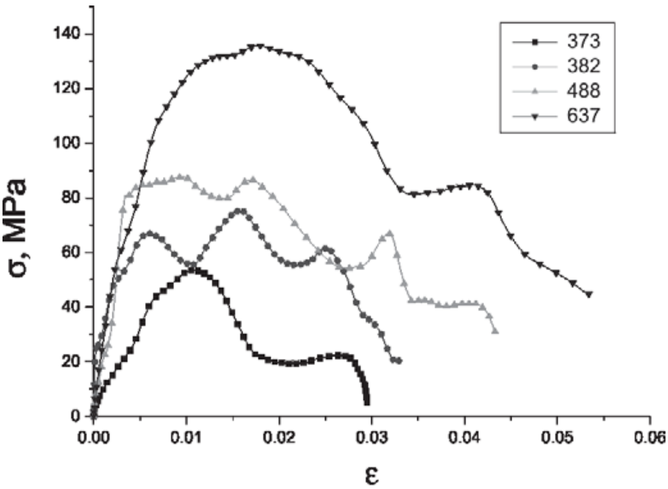

Fig. 3. Stress-strain curves for sandy mudstone in different strain rates

As the stress-strain curves for the mudstone and sandy mudstone under dynamic loads manifest significantly such dynamic mechanic characteristics as strain rigidity and plasticity, taking into account the existing dynamic mechanic models, we believe that the Zhu-Wang-Tang Model [10] can be used to describe the above-mentioned dynamic mechanic characteristics. Due to the fact that rock is a non-homogeneous material with internal defects such as grain boundaries, micro-voids, micro-cracks, weak medium and others, in the study of the damage brought about by the impacting load, the damage brought about by the internal defects of the rock should be taken into consideration. For this reason, the Zhu-Wang-Tang Model should be improved on and a new dynamic mechanic model suitable for underground soft rocks should be established.

3. The Underground Soft Rock Constitutive Model

3.1. The Zhu-Wang-Tang Model [10]

As shown in Figure 4 and presented in formula (1) below, the model is composed of two Maxwell Bodies and one nonlinear spring. One of the two Maxwell bodies is used to describe the viscoelastic response while the strain rate is low at the relaxation time φ_1. The other is used to describe the viscoelastic response while the strain rate is high at the relaxation time φ_2. And the nonlinear spring σ_E is used to describe the equilibrium stress. This model is very appropriately used to describe the dynamic mechanic behaviors of large molecule solid materials.

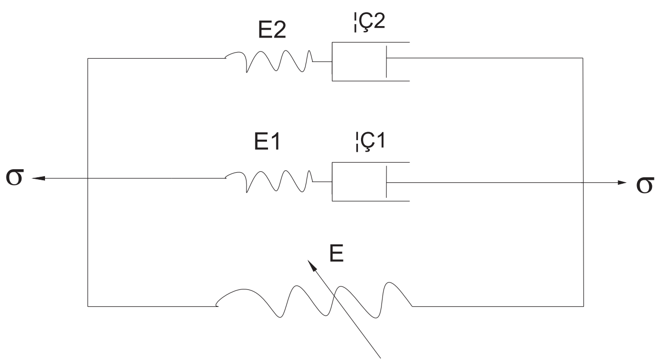

Fig. 4. Zhu-Wang-Tang model

$$\sigma(t) = (E_0\varepsilon + \beta\varepsilon^2 + \gamma\varepsilon^3) + \dot{\varepsilon} E_1\varphi_1\left(1 - e^{-\frac{\varepsilon}{\dot{\varepsilon}\varphi_1}}\right) + \dot{\varepsilon} E_2\varphi_2\left(1 - e^{-\frac{\varepsilon}{\dot{\varepsilon}\varphi_2}}\right) \tag{1}$$

3.2. The Improved Zhu-Wang-Tang Model for Study of Underground Soft Rocks

In their study of the dynamic mechanic performances of concrete, Hu Shisheng and Wang Daorong established a linear viscoelastic model [9] for the study of damage by simplifying the Zhu-Wang-Tang Model and by taking the internal damage into consideration. However, they regard the whole model as damageable, and this means that the viscous elements are damageable, implying that the physical concept is fuzzy. To overcome the defects, the present author decides to improve on the Zhu-Wang-Tang Model by substituting the damage body in the viscoelastic statistic aging damage model for the nonlinear spring in the Zhu-Wang-Tang Model, hence the introduction of a viscoelastic statistic damage model.

As shown in Figure 5, the viscoelastic statistic damage model is composed of two Maxwell Bodies and a damage body. One of the two Maxwell Bodies is used to describe the viscoelastic response while the strain rate is low at the relaxation time φ_1 and the other is used to describe the viscoelastic response while the strain rate is high at the relaxation time φ_2. In addition, the damage body is used to describe the dynamic mechanic response to the impacts on damage brought about by the internal defects of the rock when the rock is under a dynamic load.

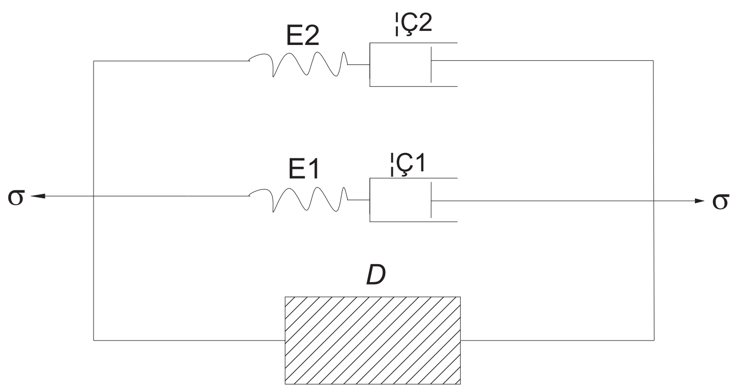

Fig. 5. Viscoelastic statistical damage model

The damage body D is regarded as linear before it is damaged. And at that time, its average elastic modulus is E_D and the strength obedience parameter is equal to the Weibull distribution of (m, α). Then for damage body D

$$D = 1 - \exp\left(-\frac{\varepsilon_D^m}{\alpha}\right),\ \varepsilon_D \geq 0 \tag{2}$$

$$\frac{\sigma_D}{1 - D} = E_D \varepsilon_D \tag{3}$$

Where ε_D the strain for the damage is body and σ_D is its stress. Then from (2) and (3), we can have

$$\sigma_D = E_D \varepsilon_D \exp\left(-\frac{\varepsilon_D^m}{\alpha}\right) \tag{4}$$

Then for φ_1 of the Maxwell Body, by series arrangement, we can have

$$\sigma_1(t) = \sigma_{11}(t) = \sigma_{12}(t) \tag{5}$$

$$\varepsilon_1(t) = \varepsilon_{11}(t) + \varepsilon_{12}(t) \tag{6}$$

Where σ_1 is the stress for the Maxwell Body at time φ_1, and σ_{11} and σ_{12} are the stresses for the elastic element and viscous element of the Maxwell Body respectively; ε_1 is the strain for the Maxwell Body at time φ_1, and ε_{11}, ε_{12} are the stresses for the elastic element and viscous element of the Maxwell Body respectively. In addition, in the formula,

$$\dot{\varepsilon}_{11}(t) = \frac{1}{E_1}\dot{\sigma}_{11}(t) \tag{7}$$

$$\dot{\varepsilon}_{12}(t) = \frac{1}{\eta_1}\sigma_{12}(t) \tag{8}$$

Where $\dot{\varepsilon}$ is the strain rate. Then by obtaining derivatives for both sides of the formula, we can have

$$\dot{\varepsilon}_1(t) = \dot{\varepsilon}_{11}(t) + \dot{\varepsilon}_{12}(t) \tag{9}$$

Then from formulae (7), (8) and (9), we can have

$$\dot{\varepsilon}_1(t) = \frac{1}{E_1}\dot{\sigma}_{11}(t) + \frac{1}{\eta_1}\sigma_{12}(t) \tag{10}$$

Then substituting formula (5) for formula (10), we can have

$$\dot{\varepsilon}_1(t) = \frac{1}{E_1}\dot{\sigma}_1(t) + \frac{1}{\eta_1}\sigma_1(t) \tag{11}$$

When the strain rate is regarded as a constant strain under load, and $\varepsilon_1(t) = \dot{\varepsilon}_1(t)t$, then let both sides of formula (11) go through Laplace transformation, we will have

$$L\left(\dot{\varepsilon}_1(t)\right) = L\left(\frac{1}{E_1}\dot{\sigma}_1(t)\right) + L\left(\frac{1}{\eta_1}\sigma_1(t)\right) = \frac{L\left(\dot{\sigma}_1(t)\right)}{E_1} + \frac{L\left(\sigma_1(t)\right)}{\eta_1},$$

$$\frac{\dot{\varepsilon}_1(t)}{S} = \frac{S\sigma_1(s) - \sigma_1(0)}{E_1} + \frac{\sigma_1(s)}{\eta_1} \tag{12}$$

Under initial conditions such as when $\sigma_1(0) = 0$, $\varepsilon_1(0) = 0$, substitute the above results for formula (12), we will have

$$\frac{\dot{\varepsilon}_1(t)}{S} = \frac{S\sigma_1(s)}{E_1} + \frac{\sigma_1(s)}{\eta_1} \tag{13}$$

By simplifying formula (13), we will have

$$\sigma_1(s) = \frac{\dot{\varepsilon}_1(t)E_1}{S\left(S + \frac{E_1}{\eta_1}\right)} \tag{14}$$

By letting the two sides of formula (14) go through Laplace transformation, we will have

$$L^{-1}\left(\sigma_1(s)\right) = L^{-1}\left(\frac{\dot{\varepsilon}_1(t)E_1}{S\left(S + \frac{E_1}{\eta_1}\right)}\right) \tag{15}$$

$$\sigma_1(t) = \dot{\varepsilon}_1(t)E_1\frac{\eta_1}{E_1}\left(1 - e^{-\frac{E_1}{\eta_1}t}\right) \tag{16}$$

By simplifying the above results, we will have:

$$\sigma_1(t) = \dot{\varepsilon}_1(t)\eta_1\left(1 - e^{-\frac{E_1}{\eta_1}t}\right) \tag{17}$$

Then by substituting $\varepsilon_1(t) = \dot{\varepsilon}_1(t)t$ for formula (15), we will have:

$$\sigma_1(t) = \dot{\varepsilon}_1(t)\eta_1\left(1 - e^{-\frac{E_1\varepsilon_1}{\eta_1\dot{\varepsilon}_1}}\right) \tag{18}$$

Then letting $\varphi_1 = \eta_1 / E_1$, formula (18) can be reduced into:

$$\sigma_1(t) = \dot{\varepsilon}_1(t)E_1\varphi_1\left(1 - e^{-\frac{\varepsilon_1}{\dot{\varepsilon}_1\varphi_1}}\right) \tag{19}$$

Similarly, the representation formula for the Maxwell Body at time φ_2 can be derived as follows:

$$\sigma_2(t) = \dot{\varepsilon}_2(t)E_2\varphi_2\left(1 - e^{-\frac{\varepsilon_2}{\dot{\varepsilon}_2\varphi_2}}\right) \tag{20}$$

Since the two Maxwell Bodies and the damage body are in series, then the following formula can be derived:

$$\sigma(t) = \sigma_D(t) + \sigma_1(t) + \sigma_2(t) \tag{21}$$

$$\varepsilon(t) = \varepsilon_D(t) = \varepsilon_1(t) = \varepsilon_2(t) \tag{22}$$

When derivation is applied to formula (20), we can have:

$$\dot{\varepsilon}(t) = \dot{\varepsilon}_D(t) = \dot{\varepsilon}_1(t) = \dot{\varepsilon}_2(t) \tag{23}$$

Then from formulae (4), (17), (18), (20) and (21), we can have:

$$\sigma(t) = E_D\varepsilon Exp\left(-\frac{\varepsilon^m}{\alpha}\right) + \dot{\varepsilon}E_1\varphi_1\left(1 - e^{-\frac{\varepsilon}{\dot{\varepsilon}\varphi_1}}\right) + \dot{\varepsilon}E_2\varphi_2\left(1 - e^{-\frac{\varepsilon}{\dot{\varepsilon}\varphi_2}}\right) \tag{24}$$

The above formula can be used to describe the stress-strain constitutive laws before the load-induced damage of underground soft rocks breaks out.

4. Application of the Constitutive Model in the Experiment

4.1. Curve Fitting and Resulted Fitting Parameters

When it reaches the ultimate stress, the rock specimen for the experiment is damaged along the crack and consequently, the stress decreases drastically. For this reason, the SHPB experiment results obtained for the strain cannot be applied to the real situation. By taking this fact into account, the present author will adopt the constitutive formula in formula (22) to fit the stress-strain curves before the ultimate stress is reached. Figures 6 and 7 show the curve fittings at different strain rates for mudstone and sandy mudstone respectively and the resulted fitting parameters are listed in Tables 1 and 2.

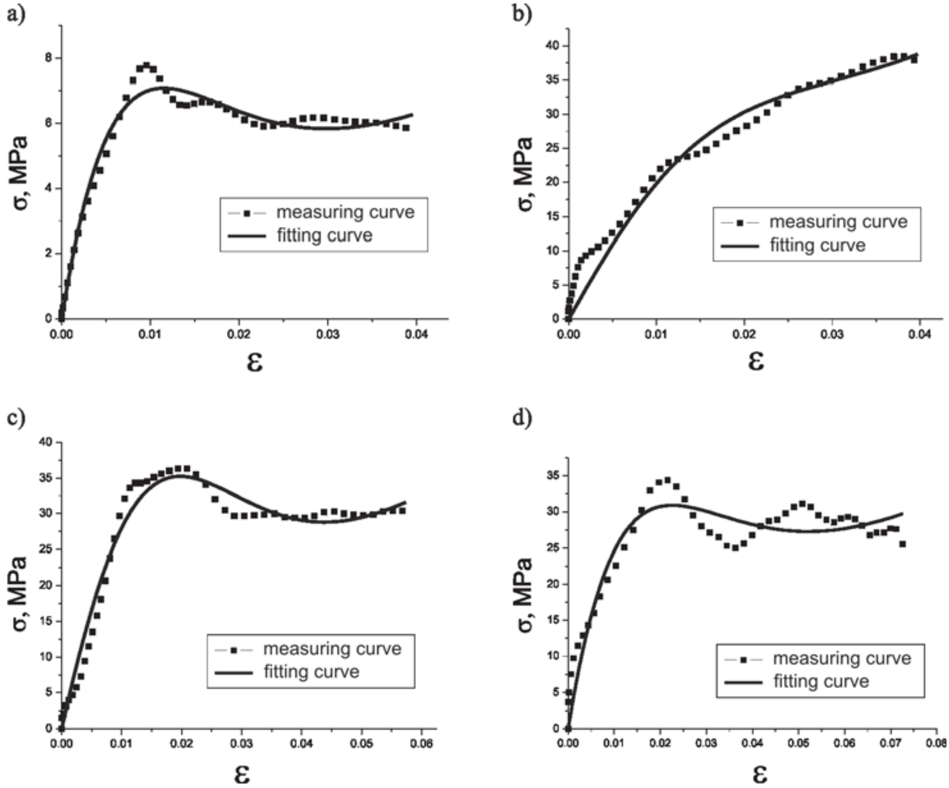

Fig. 6. Fitting curves for Mudstone in different strain rates:
a) $\dot{\varepsilon} - 551\ s^{-1}$; b) $\dot{\varepsilon} - 591\ s^{-1}$; c) $\dot{\varepsilon} - 791\ s^{-1}$; d) $\dot{\varepsilon} - 973\ s^{-1}$

As can be seen from the fitting results in Figures 6 and 7, the fitting curves and the curves obtained by measuring are basically in agreement, indicating that the model can be applied to the study of the dynamic mechanic constitutive laws for soft rocks. From various groups of the resulted fitting parameters, it can be seen that the parameter m varies in value around 1.5 and when the mth power of α is obtained, the root varies around the yield strain corresponding to the yield stress.

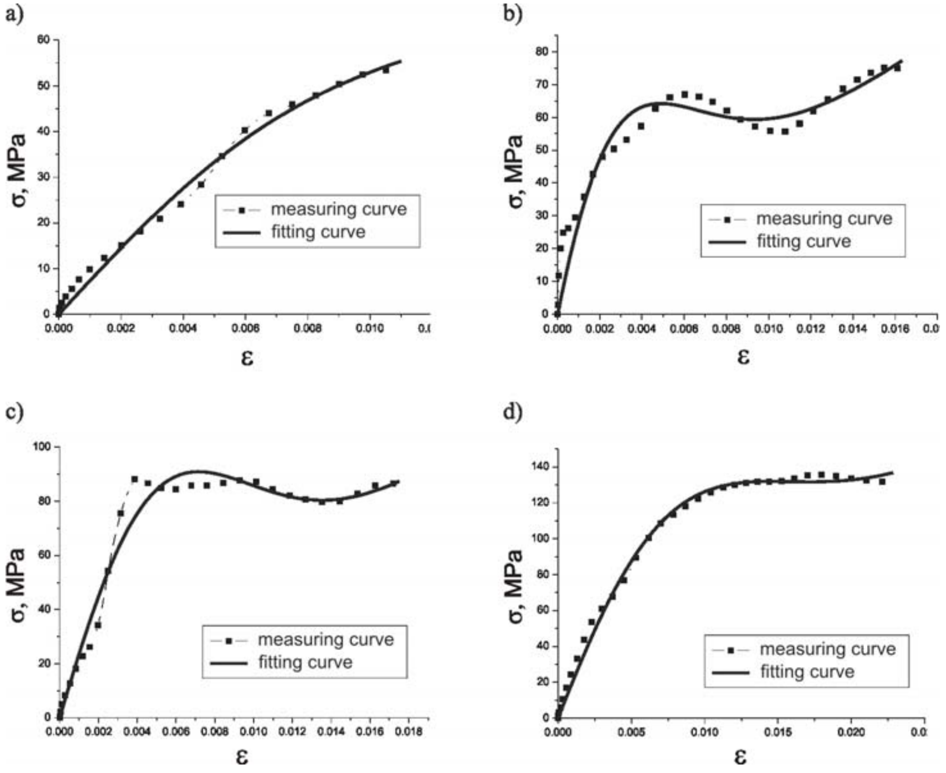

Fig. 7. Fitting curves for sandy mudstone in different strain rates:
a) $\dot{\varepsilon}$ – 373 s^{-1}; b) $\dot{\varepsilon}$ – 382 s^{-1}; c) $\dot{\varepsilon}$ – 488 s^{-1}; d) $\dot{\varepsilon}$ – 673 s^{-1}

The relaxation times for mudstone and sandy mudstone are not constants. The responses from the two stones to low strain rates at the relaxation time φ_1 are not much different from the response from the concrete [9] or the response from anthracite [11]. This shows that the responses from rocks, concrete and anthracite to low strain rate are in agreement. However, to high strain rates at the relaxation time φ_2, the responses from the two stones will be different from those from concrete and anthracite, indicating that the two stones, concrete and anthracite are sensitive to high strain rate. When it comes to the two stones, while at the relaxation time φ_1, mudstone and sandy mudstone are insignificantly differently sensitive to low strain rate, the response from mudstone at the relaxation time φ_2 is greater in value than the response from sandy mudstone at the same relaxation time, indicating that mudstone is more sensitive to high strain rate than sandy mudstone.

TABLE 1

Fitting parameters for Mudstone in different strain rates

Parameter Fig.	Strain rate, s^{-1}	E_D, GPa	E_1, GPa	φ_1, s^{-1}	E_2, GPa	φ_2, s^{-1}	m	α, 10^{-4}
(a)	551	1.54	0.13	0.37	0.13	8.7	1.10	6.11
(b)	591	1.42	0.29	0.37	0.57	8.7	1.73	1.49
(c)	791	3.10	0.49	0.37	0.03	8.7	1.57	2.29
(d)	973	3.49	0.36	0.37	0.013	8.7	1.10	12.54

TABLE 2

Fitting parameters for sandy mudstone in different strain rates

Parameter Fig.	Strain rate, s^{-1}	E_D, GPa	E_1, GPa	φ_1, s^{-1}	E_2, GPa	φ_2, s^{-1}	m	α, 10^{-4}
(a)	373	4.01	3.14	0.3	0.23	4.5	1.88	2.3
(b)	382	26.61	1.33	0.3	3.32	4.5	1.36	6.4
(c)	488	19.20	0.68	0.3	4.08	4.5	1.81	1.5
(d)	673	15.45	4.85	0.3	0.35	4.5	1.69	5.6

5. Conclusion

1) Under dynamic load, mudstone and sandy mudstone will be characterized by dynamic mechanic features such as rigidity and plastic flexibility. Before the stress-strain curve reaches its yield stress stage, mudstone and sandy mudstone are at the stage of plastic deformation. The deformation is nonlinear. The lower the strain, the more apparently linear the deformation. Generally, the plastic modulus for mudstone increases with the increase of the strain rate, indicating that the strain rate serves as a function of the plastic modulus.
2) Within a certain strain range, the damage strain of the stone is in proportion with the increase of strain rate. However, beyond the range, the strengthening of strain will exercise insignificant impacts on the dynamic damage of the stone.
3) The viscoelastic statistic damage model for the damage body can be satisfactorily employed to describe the response from sandy mudstone to either high or low strain rate. In addition, the fitting curve is in agreement with the curve obtained by measurement.
4) The two stones are in agreement in their responses to low strain rates and they are sensitive to high strain rates. In addition, mudstone responds more sensitive to high strain rate than sandy mudstone.

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