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A STATIC MODEL FOR SELECTING TECHNICAL FACILITIES OF A LOGISTICS CENTRE BASED ON THE TASKS TO BE PERFORMED

Abstract: This paper discusses a method of optimisation of technical facilities of logistics systems. The presented approach is based on a static model of cargo flows distribution among the structural elements of the analysed system. The method has been verified using a real logistics centre as an example.

Keywords: facility optimisation, logistics centre, static model of cargo flow.

1. Introduction

The efficiency of logistics processes continues to gain in importance, as companies search for new possibilities to increase their competitive edge and business potential, and thus improve effectiveness. The changes in transport and shipment accompanied by rapid growth in the logistics services market provide new conditions for functioning of enterprises.

The development of logistics services is inseparable from the development of various logistics complexes, such as distribution centres, regional depots, logistics centres, etc. The quality requirements for such complexes reflect the need for comprehensive services, including handling, storage and commissioning. Therefore, any logistics systems to be developed should be subject to a thorough analysis aimed at selecting the technical infrastructure and facilities suitable for the logistics tasks to be performed.

A logistics task is the key determinant of the logistics system design. It determines the required scope of transformations of physical cargo flows and the related information flows. Hence, it is essential to seek methods for optimising the technical facilities of logistics systems to make them fit to changing logistics tasks.

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2. Logistics system model

A primary tool for the analysis and evaluation of logistics systems, either existing or newly designed, is a model of such a system. The key objective of logistics systems is to assure efficient cargo flows and the related information flows. Hence, a model of a logistics system (LS) should map the following properties:

- *structure*: representing connections between LS elements;
- *volume of logistics tasks*: LS loading with cargo flows;
- *characteristics of structural elements*: representing actual properties of LS structural elements that are important in terms of facilities selection; and
- *organisation*: representing links between LS facilities and the cargo flows handled.

Let $\boldsymbol{G}$ be a structure mapping, $\boldsymbol{Q}$ be the volume of logistics tasks, $\boldsymbol{F}_G$ be the set of characteristics of LS structural elements and $\boldsymbol{O}$ be the LS organisation; then, the **L**ogistics **S**ystem **M**odel (LSM) may be defined as an ordered four:

$$LSM = \langle \boldsymbol{G}, \boldsymbol{Q}, \boldsymbol{F}_G, \boldsymbol{O} \rangle \tag{1}$$

where:

$\boldsymbol{G}$ – logistics system structure graph,
$\boldsymbol{Q}$ – volume (size) of logistics tasks (cargo flows handled by LS),
$\boldsymbol{F}_G$ – set of functions defined over LS elements,
$\boldsymbol{O}$ – LS organisation (which determines how cargoes flow through the system).

Taking into account the above notation, for the purpose of LS modelling for facilities selection, the structure of a logistics system may be represented as a graph $\boldsymbol{G}$:

$$\boldsymbol{G} = \langle \boldsymbol{W}, \boldsymbol{L} \rangle \tag{2}$$

where:

$\boldsymbol{W}$ – logistics system structure graph,
$\boldsymbol{L}$ – volume (size) of logistics tasks (cargo flows handled by LS).

Hence, the set $\boldsymbol{W}$ of logistics system elements is defined as follows:

$$\boldsymbol{W} = \{1, 2, ..., i, j, ..., I\} \tag{3}$$

Furthermore, it is assumed that the set of LS elements always contains the following subsets:

- Set $\boldsymbol{A}$ of cargo flow sources or system inputs:

$$\boldsymbol{A} = \{i : a(i) = 1, i \in \boldsymbol{W}\}.$$

– Set $\boldsymbol{B}$ of cargo flow outlets or system outputs:

$$\boldsymbol{B} = \{i : b(i) = 1, i \in \boldsymbol{W}\}.$$

– Set $\boldsymbol{V}$ of logistics system elements identified as LS technical facilities:

$$\boldsymbol{V} = \{i : a(i) = 0 \wedge b(i) = 0, i \in \boldsymbol{W}\}.$$

In consideration of the foregoing, the set of LS elements may be decomposed into three subsets, namely set $\boldsymbol{A}$, set $\boldsymbol{B}$ and set $\boldsymbol{V}$, which meet the following condition:

$$\boldsymbol{W} = \boldsymbol{A} \cup \boldsymbol{V} \cup \boldsymbol{B} \text{ and } \boldsymbol{A} \cap \boldsymbol{V} \cap \boldsymbol{B} = \emptyset$$

It is also assumed that LS facilities may include various types of warehouses (regional depots) (**WH**), parking lots (**PL**), storage yards (**SY**), on-site transport systems (**ST**) or reloading facilities (**RF**) (see Fig. 1).

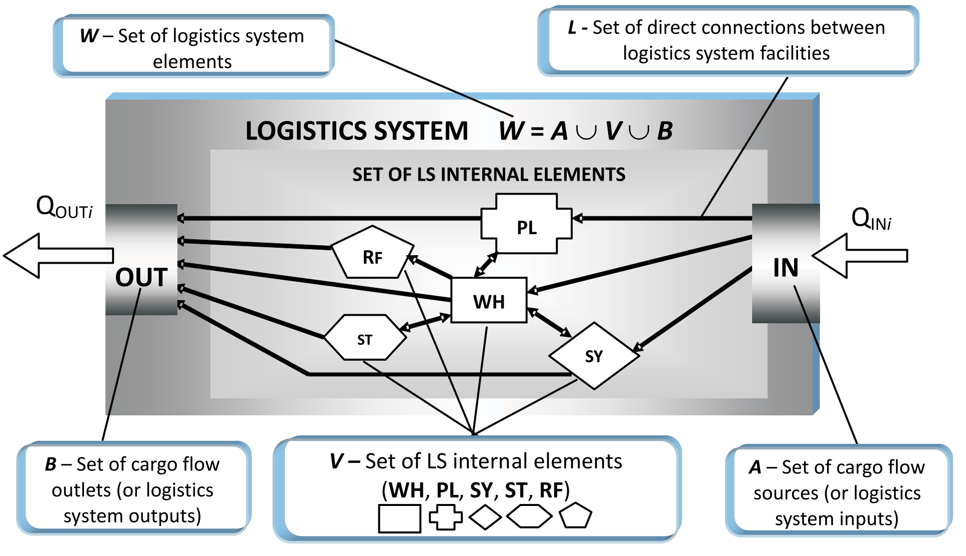

Fig. 1. *Logistics System Structure: Q_{INi} – Cargo flow at a logistics system input, Q_{OUTi} – Cargo flow at a logistics system output*

Furthermore, the set $\boldsymbol{L}$ of LS structure graph edges, mapping the existing direct connections between logistics system elements, is defined as follows:

$$\boldsymbol{L} = \{(i,j) : \xi(i,j) = l; i,j \in \boldsymbol{W}, i \neq j\}$$

where ξ is a mapping defined over the Cartesian product $\boldsymbol{W} \times \boldsymbol{W}$.

For the purpose of modelling aimed at selecting LS technical facilities suitable for the specific tasks, a logistics task may be identified by the following:

– Set $\boldsymbol{R}$ of cargo types handled by the logistics system:

$$\boldsymbol{R} = \{1, 2, ..., r, ..., R\}$$

– LS load with logistics tasks represented by the vector $\boldsymbol{Q}$:

$$Q = [q1(r) \in \boldsymbol{\mathcal{R}}^+ : r \in \boldsymbol{R}]$$

– Matrix $\boldsymbol{QZ}$ of volumes of different cargo flows arriving at LS inputs:

$$QZ = [q2(i, r) \in \boldsymbol{\mathcal{R}}^+ : i \in \boldsymbol{A},\ r \in \boldsymbol{R}]$$

Furthermore, let us define a set $\boldsymbol{R}(i)$ of numbers corresponding to different types of cargoes that may be handled by the particular LS structure element:

$$\boldsymbol{R}(i) = \{r : \gamma(i, r) = \ 1, r \in \boldsymbol{R}\},\ i \in \boldsymbol{W}$$

Cargo flow transformations require various types of facilities. Hence, it has been assumed that each logistics system facility is described by its type. The set $\boldsymbol{TU}(i)$ of all types of facilities for the logistics system element i is defined as follows:

$$\boldsymbol{TU}(i) = \{1, 2, ..., tu(i), ..., TU(i)\},\ i \in \boldsymbol{W}$$

For the purpose of representing the properties of logistics system structure elements, it has been assumed that functions defined over LS structure elements may describe their different characteristics, including technical (e.g. cargo flow service time, throughput, capacity, etc.), economic (e.g. cargo flow service cost, capital expenditure required for the particular LS element) or mathematical (e.g. probability of transition between LS elements).

There is no requirement for all the subsets of the set of characteristics defined over LS elements to include the aforementioned elements. However, at least one of these subsets must be non-empty.

In consideration of the above, the set of characteristics of logistics system elements, $\boldsymbol{F}_W$, and the set of characteristics of transport connections between logistics system elements, $\boldsymbol{F}_L$, have been defined.

It has been assumed that the set $\boldsymbol{F}_W$ is the sum of:

– The set $\boldsymbol{F}_{VV}$ of functions defined over the set $\boldsymbol{V}$, i.e.:

$$\boldsymbol{F}_{VV} = \{f_w(i) \in \boldsymbol{\mathcal{R}}^+ : i \in \boldsymbol{V};\ \ w = 1, 2, \ldots, W\};$$

– The set $\boldsymbol{F}_{VN}$ of functions defined over the set of types of facilities for individual LS elements, i.e.:

$$\boldsymbol{F}_{VN} = \{f_{wn}(i, tu(i)) \in \boldsymbol{\mathcal{R}}^+ : i \in \boldsymbol{V}; tu(i) \in \boldsymbol{TU}(i); wn = 1, 2, ..., WN\};\ \text{and}$$

– The set $\boldsymbol{F}_{VR}$ of functions defined over the Cartesian product $\boldsymbol{TU}(i) \times \boldsymbol{R}$ for various facilities at LS elements, i.e.:

$$\boldsymbol{F}_{VR} = \{f_{vr}(i, tu(i), r) \in \boldsymbol{\mathcal{R}}^{+} : i \in \boldsymbol{V}; tu(i) \in \boldsymbol{TU}(i); r \in \boldsymbol{R}; vr = 1, 2, ..., VR\}.$$

Furthermore, the set $\boldsymbol{F}_L$ is the sum of:

– The set $\boldsymbol{F}_{LN}$ of functions defined over the set $\boldsymbol{L}$, i.e.:

$$\boldsymbol{F}_{LN} = \{f^{l}(i, j) \in \boldsymbol{\mathcal{R}}^{+} : (i, j) \in L; l = 1, 2, ..., L\}; \text{ and}$$

– The set $\boldsymbol{F}_{LR}$ of functions defined over the Cartesian product $\boldsymbol{L} \times \boldsymbol{R}$, i.e.:

$$\boldsymbol{F}_{LR} = \{f^{lr}((i, j), r) \in \boldsymbol{\mathcal{R}}^{+} : (i, j) \in \boldsymbol{L}; r \in \boldsymbol{R}; lr = 1, 2, ..., LR\}.$$

Hence, the set of all functions defined over structure elements of the particular logistics system, $\boldsymbol{F}_G$, is the sum of the set of characteristics of the elements thereof, $\boldsymbol{F}_W$, and the set of functions defined over the set of direct connections between the elements thereof, $\boldsymbol{F}_L$.

3. Optimisation problems related to selection of logistics system facilities

In any optimisation problem of selecting logistics system technical facilities that will assure efficient performance of the LS tasks, the key problem is to determine the quantity of LS elements as well as their loading with the tasks performed. The underlying assumption is that all logistics tasks arriving at the system in the given period have to be performed.

While determining the quantity of various types of facilities within the analysed logistics system as well as their loading with cargo flows, one has to account for various constraints, particularly resulting from:

– requirement to perform the specific logistics tasks,
– continuity of cargo flow through the logistics system,
– limited capacity (throughput) of logistics system facilities,
– conditions imposed on cargo flows (e.g. non-negativity),
– limits for service time, cost of performance, etc.

Naturally, these constraints reflect partly the very nature of the system and partly user expectations. If the aforementioned constraints are met, a feasible solution may be found.

In order to identify the optimum facilities of the particular logistics system that will assure effective performance of the pre-defined logistics tasks, one has to define the criteria for the evaluation of different solutions.

For the purposes of formulating the relevant optimisation problem, it has been assumed that the following characteristics of logistics system facilities are given:

- total capital expenditure on the facility of the type $tu(i)$ at the logistics system element i, denoted as $f^1(i, tu(i)) \equiv n(i, tu(i))$;
- capital expenditure on the facility $tu(i)$ at the logistics system element i plus fixed costs in the analysed period T, denoted as $f^2(i, tu(i)) \equiv ks(i, tu(i))$;
- variable costs of operation of the facility $tu(i)$ at the logistics system element i per unit of cargo flow of type r, denoted as $f^3(i, tu(i), r) \equiv kz(i, tu(i), r)$;
- service time for cargo flow of type r handled by the facility $tu(i)$ at the logistics system element i, denoted as $f^4(i, tu(i), r) \equiv \tau(i, tu(i), r)$;
- load non-uniformity factor for the logistics system element i, denoted as $f^5(i) \equiv \varphi(i)$;
- the probability of cargo transition between the elements i and j, denoted as $f^6(i, j) \equiv p(i, j)$.

In consideration of the above, an optimisation problem related to selecting technical facilities for a particular logistics system intended to perform the predefined logistics tasks may be formulated as follows:

For the following input data:

$\boldsymbol{G}$ – logistics system structure graph,
$\boldsymbol{F}_V$ – set of functions defined over LS elements:

$$\boldsymbol{F}_V = \{n(i, tu(i)), ks(i, tu(i)), kz(i, tu(i), r), \tau(i, tu(i), r), \varphi(i) : \\ i \in \boldsymbol{V}, tu(i) \in \boldsymbol{TU}(i), r \in \boldsymbol{R}(i)\} \tag{4}$$

$\boldsymbol{F}_L$ – set of functions defined over connections between LS elements:

$$\boldsymbol{F}_L = \{p(i, j) : \xi(i, j) = 1, i \in \boldsymbol{A} \cup \boldsymbol{V}, j \in \boldsymbol{V} \cup \boldsymbol{B}\} \tag{5}$$

$\boldsymbol{QZ}$ – matrix of cargo flow volumes at system inputs,
$\boldsymbol{T}$ – time of operation of the logistics system,
$\boldsymbol{CX}$ – capital expenditure (on the LS construction or modernisation);

find the values of the decision variables:

$\boldsymbol{X}$ – vector of types of facilities for individual LS elements,

$$\boldsymbol{X} = [x(i, tu(i)) \in \boldsymbol{\mathcal{N}} : \ i \in \boldsymbol{V} \ , tu(i) \in \boldsymbol{TU}(i)] \tag{6}$$

$\boldsymbol{Y}$ – matrix of cargo flow loads on different types of LS facilities:

$$\boldsymbol{Y} = [y(i, tu(i), r) \in \boldsymbol{\mathcal{R}}^+ : i \in \boldsymbol{V}, tu(i) \in \boldsymbol{TU}(i), r \in \boldsymbol{R}(i)] \tag{7}$$

which meet the following constraints:

– Performance of all the logistics tasks arriving at LS in the given period, i.e.:

$$\sum_{tu(j)\in \boldsymbol{TU}(j)} y(j,tu(j),r) = q2(i,r)\cdot p((i,j),r), \quad i\in \boldsymbol{A}, r\in \boldsymbol{R}(i), j\in \Gamma(i,r) \tag{8}$$

– Continuity of cargo flows through LS, i.e.:

$$\begin{aligned}&\sum_{j\in\Gamma(i,r)}\sum_{tu(i)\in \boldsymbol{TU}(i)} y(i,tu(i),r)\cdot p((i,j),r) =\\ &= \sum_{tu(j)\in \boldsymbol{TU}(j)} y(j,tu(j),r),\ i\in \boldsymbol{V},\ r\in \boldsymbol{R}(i)\end{aligned} \tag{9}$$

– Capacity of LS facilities, i.e.:

$$\sum_{r\in \boldsymbol{R}(i)} y(i,tu(i),r)\cdot\tau(i,tu(i),r) \leqslant \frac{T}{\varphi(i)}\cdot x(i,tu(i)), \quad i\in \boldsymbol{V}, tu(i)\in \boldsymbol{TU}(i) \tag{10}$$

– Limits for capital expenditure on LS facilities, i.e.:

$$\sum_{i\in \boldsymbol{V}}\sum_{tu(i)\in \boldsymbol{TU}(i)} n(i,tu(i))\cdot x(i,tu(i)) \leqslant NI \tag{11}$$

– Integer number of pieces of equipment within LS, i.e.:

$$x(i,tu(i))\in \boldsymbol{\mathcal{N}}, \quad i\in \boldsymbol{V}, tu(i)\in \boldsymbol{TU}(i),\ \text{and} \tag{12}$$

– Non-negative loads on LS facilities i.e.:

$$y(i,tu(i),r)\geqslant 0, \quad i\in \boldsymbol{V}, tu(i)\in \boldsymbol{TU}(i), r\in \boldsymbol{R}(i) \tag{13}$$

to minimise the objective function, interpreted as:

– Time of performance of the logistics tasks:

$$k1(Y) = \sum_{i\in \boldsymbol{V}}\sum_{tu(i)\in \boldsymbol{TU}(i)}\sum_{r\in \boldsymbol{R}(i)} \tau(i,tu(i),r)\cdot y(i,tu(i),r) \to \min,\ \text{or} \tag{14}$$

– Cost of performance of the logistics tasks:

$$k2(X,Y) = \sum_{i\in \boldsymbol{V}}\sum_{tu(i)\in \boldsymbol{TU}(i)} \begin{pmatrix} ks(i,tu(i))\cdot x(i,tu(i)) + \\ + \sum_{r\in \boldsymbol{R}(i)} kz(i,tu(i),r)\cdot y(i,tu(i),r)\end{pmatrix} \tag{15}$$

The optimisation problem of selecting LS technical facilities that will assure performance of the predefined logistics tasks presented above is a single-objective problem.

If both the criteria formulated above are interpreted as partial objectives, than a multi-objective optimisation problem may be formulated. In general, a two-objective optimisation problem of selecting LS technical facilities for the performance of specific logistics tasks may be presented as follows:

For the input data:	$\boldsymbol{G} = \langle \boldsymbol{W}, \boldsymbol{L} \rangle, \boldsymbol{F}_V, \boldsymbol{F}_L, \boldsymbol{QZ}, T, \boldsymbol{CX}$
find:	$\boldsymbol{X} = [x(i, tu(i))], \quad \boldsymbol{Y} = [y(i, tu(i), r)]$
that meet the constraints:	$(5 \div 10)$
to optimise the objective function:	$\boldsymbol{K}(\boldsymbol{X}, \boldsymbol{Y}) = [k1(\boldsymbol{Y}); k2(\boldsymbol{X}, \boldsymbol{Y})]$

4. Selection of logistics system facilities (using a logistics centre as an example)

To illustrate the approach presented above, some computational experiments were conducted for a real logistics centre (LC). The LC in question is supplied by 5 suppliers and ships goods to 7 customers[1].

On the basis of the available data, it was established that the centre received the following volumes of cargo annually:

– $94,558,000$ Mt of industrial goods for footwear production (e.g. leather, rubber, etc.);
– $236,540,000$ Mt of group I foodstuff (e.g. hazelnuts, raisins, etc.):
– $271,936,000$ Mt of ingredients for food production (e.g. cocoa, milk powder, etc.);
– $89,618,000$ Mt of group II foodstuff (e.g. sweets: wafer biscuits and chocolate);
– $75,685,000$ Mt of footwear (e.g. sport shoes for young people).

The following elements of the LC structure (Fig. 1) are used for handling palletised or container cargo units:

– high-storage warehouse with a completion zone,
– loading terminal with a container storage yard,
– parking lots in which trucks can wait for service (waiting for a free docking ramp or a travelling crane system),
– truck entrance/exit gates,
– train entrance/exit gates,
– railroad and road systems,
– site route system.

[1] The data of the suppliers and customers as well as other detailed data on cargo flows throught the TOMAZ centre are contained in the thesis.

The purpose of a functional analysis of the logistics centre was to determine the best option of technical facilities. The analysis was based on the following assumptions:

- the container terminal may have the following facilities: gantry cranes, moving cranes;
- the storage zone facilities include mainly high rack systems, operated by rack stackers, and medium rack systems (3–4 levels), operated by forklift trucks;
- the delivery/dispatch zone facilities include mainly forklift trucks for horizontal transport;
- docking ramp facilities include loading docks and forklift trucks;
- completion zone facilities include forklift trucks for order completion;
- the logistics centre has arrival/departure tracks and siding tracks for handling railway transport;
- for entrance/exit gates, the work posts related to their operation were considered as "facilities".

The analysis of the LC facilities was conducted for one year, i.e. $T = 374,400$ min[2]. Based on the available data, the structure elements characteristics required for solving the problem were determined, namely:

- volume of different cargo flows arriving at system inputs, in units (pieces) per year;
- period of analysis of LC facilities: one year, i.e. $T = 374,400$ min[3];
- load non-uniformity factors, $\varphi(i)$, for particular LC facilities;
- service times, in minutes, per unit of cargo handled by various facilities at LC elements;
- unit variable costs of operation of various facilities at LC elements, in PLN;
- capital expenditure and fixed costs;
- capital expenditure, in PLN '000, on particular logistics centre facilities.

Taking into account the assumptions concerning the input data and the set of constraints (5 – 10), the relevant optimisation problem was formulated. The decision variables were the vector of the number of facilities installed in the logistics centre, $\boldsymbol{X}$, and the matrix of facility loads, $\boldsymbol{Y}$. The criterion for evaluating the problem solutions, i.e. the objective, was the cost of performance of the tasks.

Then, the WinQsb application was used to find the solution of the formulated problem of selecting technical facilities of the logistics centre analysed.

The analysis results indicated that travelling cranes should be chosen from among different options of container handling systems. The decisive factor was definitely the related costs. In addition, the number of truck docking stations at the warehouse turned out to be insufficient.

[2] For three shifts and 260 working days.
[3] For three shifts and 260 working days.

5. Conclusions

The approach presented in the paper is suitable for solving a wide range of problems related to selecting technical facilities to perform the predefined logistics tasks.

In particular, it can be applied to:

- an analysis of whether the logistics centre facilities are suitable for the existing or intended loads;
- studies by companies which want to provide optimal facilities within their own storage and transport systems.

The cargo flows may be analysed and assessed for both the existing and newly designed logistics systems. Hence, the presented approach may constitute a supporting tool in making decisions related to selection of the infrastructure and technical facilities of logistics centres that offer various scope of services.

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