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ABOUT ONE PROBLEM OF THE NONISOTHERMAL FILTRATION PROCESS

As the sizes of times real deposit structures are within the limits of several shares of micron it is possible to consider speed of such exchanges infinitely fast for thermo-physically homogeneous areas. Thus numerical experiments show, that temperature the front usually lags behind front of a saturation and the case of warming up of an oil layer $\theta_0 > \theta_p$ increases only a degree of final petrofeedback. At calculations it is shown in the form of occurrence of additional front of replacement (area of greater gradients of a pressure), corresponding thermal front (area of greater gradients of temperature) that meets results in works of other authors [1–3].

With another, it is known, that in a reality there are cases when structure and a structure of times of a layer homogeneously (porosity and permeability are constant), but the layer consists of various breeds with different thermo-physically properties. There is a question on structure of movement of a liquid in a collector and algorithms of the decision of the given problems. Below one of approaches of modelling of similar processes when the full charge of phases $v + v_1 = V(t)$ is set is resulted.

Let's consider not isothermal current of a liquid in the isotropic porous environment in view of processes of heat exchange on the basis of mathematical model of Bakley-Leverette, being limited to a case of one-dimensional movement at known value of total speed of phases [4, 5]:

$$v_i = -\frac{K_0}{\mu_i} f_i(s) \frac{\partial p}{\partial x},$$
$$m\frac{\partial s}{\partial t} + \frac{\partial v}{\partial x} = 0, \quad -m\frac{\partial s}{\partial t} + \frac{\partial v_1}{\partial x} = 0, \tag{1}$$

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At set $v+v_1=V(t)$ the equation for a petrosaturation will be transformed, as

$$m\frac{\partial s}{\partial t}+V(t)F_s'\frac{\partial s}{\partial x}=0 \qquad (2)$$

Which has the common decision of a kind [3, 4]

$$X(s,\tau)=\tau F'(s)+X^0(s) \qquad (3)$$

Where:

$X,\ \tau=\frac{1}{m}\int_0^t V(t)dt$ – the transformed independent variables;

s – oil saturation;

$s_1=1-s$ – a water saturation;

$F'(s)$ – derivative of Leverette;

$X^0(s)$ – distribution of an initial oil saturation.

Including the layer saturated by fluids heterogeneous structure, heat exchange between elements of this structure we shall present the equation of kinetics a kind

$$\alpha_T\frac{\partial\theta_p}{\partial t}=\theta_0-\theta_p \qquad (4)$$

Where:

α_T – small parameter of kinetics;

θ_p – temperature of a skeleton of the porous environment and it is possible, together with the motionless liquids connected with it

θ_0 – temperature in mobile fluids.

Heat transfer, carried out by these fluids (water and oil) it is described by the equation [6]:

$$\frac{\partial}{\partial t}\{b(s)\cdot\theta+c(s)\}=\frac{\partial}{\partial x}\left(\lambda(s)\cdot\frac{\partial\theta}{\partial x}-(b(s)\cdot\theta+c(s))V(t)\right) \qquad (5)$$

Connected with a saturation a ratio

$$s(x,t)=\chi(\theta)=\begin{cases} s^*,\ \theta<\theta^* \\ (s_*,s^*),\ \theta=\theta^* \\ s_*,\ \theta>\theta^* \end{cases} \qquad (6)$$

$$s+s_1=1$$

Here $b = b(s), c = c(s), \lambda = \lambda(s) = \lambda_{oil} \cdot m \cdot s + \lambda_{water} \cdot m \cdot (1-s) + \lambda_{pormedia} \cdot (1-m)$.
Boundary and entry conditions enter the name in the form of:

$$
\begin{aligned}
& s\big|_{t=0} = s_0, \quad s(0,t) = \bar{s}, \quad \left.\frac{\partial s}{\partial x}\right|_{x=1} = 0, \\
& v + v_1 = V(t), \\
& \theta\big|_{t=0} = \theta^0, \quad \theta(0,t) = \theta_0, \quad \theta(1,t) = \theta_p \\
& [\mathrm{s}]\big|_{x=R^{\pm}(t)} \neq 0, \quad [\mathrm{v}]\big|_{x=R^{\pm}(t)} = 0, \quad [\mathrm{s}]\big|_{x=H(t)} \neq 0.
\end{aligned}
\tag{7}
$$

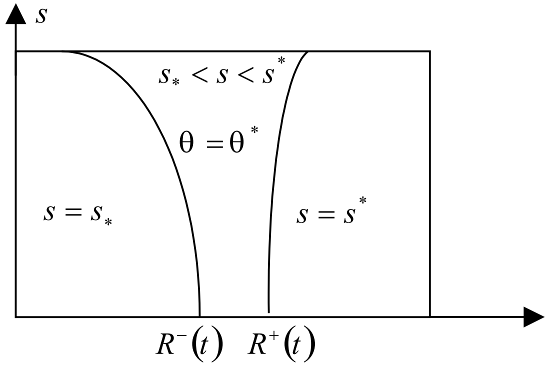

Fig. 1. The scheme of process

Thus it is necessary to consider a necessary condition for mobile borders $R^-(0) = R^+(0) = x_0$ and $R^-(t) < R^+(t)$ at $t > 0$. The general scheme of process is shown in Figure 1. It is natural, that in the given statement various cases of course of processes are probable, but thus it is necessary to consider presence of double front: replacement $H(t)$ and thermal $R^-(t)$, $R^+(t)$. As, change first of all value water pressure and temperatures are determined with final distribution of values of a petrosaturation in a layer. Following cases are possible:

1. $R^-(t) < R^+(t) < H(t)$

2. $R^-(t) < H(t) \leq R^+(t)$

Depending on these cases it is received structurally various structures for required parameters of a problem so both of a variant correspond to a case when structure and a structure of times of a layer homogeneously (porosity and permeability are constant), but the layer consists of various breeds with different thermo-physically properties. For full adequate short circuit of the equations it is necessary to define x_0 for the beginning, s^* and s_*.

Then by analogy to the approach described in works [2–3] it is found for average on area of replacement of a petrosaturation:

$$< s(\alpha) >= \frac{1}{x_f}\int_0^{x_f} s(x,t)dx = s_f - \frac{F(s_f)}{F'(s_f)}, \quad < s(\alpha)^- >= \frac{1}{x_T}\int_0^{x_T} s(x,t)dx = s_T^- - \frac{F_-(s_T^-)}{F'_-(s_T^-)} \tag{8}$$

$$< s(\alpha)^+ >= \frac{1}{x_f - x_T}\int_{x_T}^{x_f} s(x,t)dx = \frac{s_f F'_+(s_f) - s_T^+ F'_+(s_T^+) - F_+(s_f) + F_+(s_T^+)}{F'_+(s_f) - F'_+(s_T^+)}$$

Last parity defines value x_0, i.e. $x_0 = x_T$ $s^* =< s^+ >$, $s_* =< s^- >$ and $H(t) = x_f$, that allows to receive the first approach an initial problem and most adequately to describe investigated process.

Further we shall in detail disassemble a variant when in (4) speed of aspiration $\alpha_T \neq 0$ and $R^-(t) < R^+(t) < H(t)$, whence we shall consider separately cases:

1. $s = s_*$

2. $\theta = \theta^*$

3. $s = s^*$

When $s = s_*$:

$$\frac{\partial}{\partial t}\{b(s_*)\cdot\theta + c(s_*)\} = \frac{\partial}{\partial x}\left(\lambda(s_*)\cdot\frac{\partial\theta}{\partial x} - (b(s_*)\cdot\theta + c(s_*))V(t)\right)$$

Whence

$$b(s_*)\cdot\frac{\partial\theta}{\partial t} = \lambda(s_*)\cdot\frac{\partial^2\theta}{\partial x^2} - b(s_*)V(t)\cdot\frac{\partial\theta}{\partial x} \tag{9}$$

By analogy in works [7] it is defined boundary and entry conditions in the form of:

$$\theta\big|_{t=0} = \theta^0, \quad \theta(0,t) = \theta_0, \quad \theta(R^-,t) = \theta^*, \quad R^-(0) = x_0$$

$$F^{\therefore} = F^* \quad \text{or} \quad \frac{\alpha^{\therefore} f(s^{\therefore})}{\alpha^{\therefore} f(s^{\therefore}) + f_1(s^{\therefore})} = \frac{\alpha_* f(s_*)}{\alpha_* f(s_*) + f_1(s_*)} \tag{10}$$

$$[b(s^{\therefore})\theta^* + c(s^{\therefore}) - b(s_*)\theta(R^- - 0,t) - c(s_*)](\frac{dR^-}{dt} + V(t)) = -\lambda^-(s_*)\cdot\frac{\partial\theta(R^- - 0,t)}{\partial x},$$

Where $s^{\because} \in \{(s_*, s^*): x = R^- + 0\}$, which it is searched from a penultimate ratio, and last expression from (10) allows to write out iterative process for a finding R^-:

$$R^{-(n+1)} = R^{-(n)} - \tau_{it}(\overline{B}^n \cdot \left[\frac{\partial\theta(R^- - 0, t)}{\partial x}\right]^n + V(t)^n).$$

The scheme of a choice of parameters and conditions of process is shown in Figure 2.

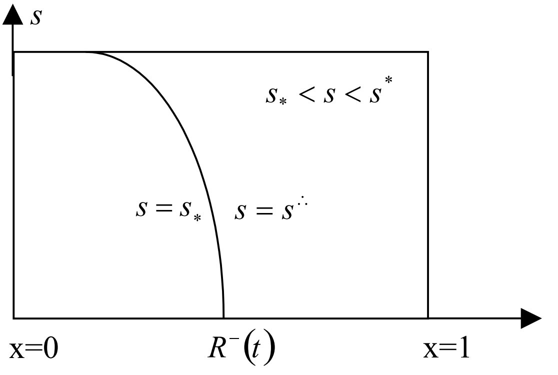

Fig. 2. Process in border R^-

When $s = s^*$:

$$\frac{\partial}{\partial t}\{b(s^*)\cdot\theta + c(s^*)\} = \frac{\partial}{\partial x}\left(\lambda(s^*)\cdot\frac{\partial\theta}{\partial x} - (b(s^*)\cdot\theta + c(s^*))V(t)\right)$$

Whence

$$b(s^*)\cdot\frac{\partial\theta}{\partial t} = \lambda(s^*)\cdot\frac{\partial^2\theta}{\partial x^2} - b(s^*)V(t)\cdot\frac{\partial\theta}{\partial x} \tag{11}$$

$$\theta\big|_{t=0} = \theta^0, \quad \theta(1,t) = \theta_p, \quad \theta(R^+, t) = \theta^*, \quad R^+(0) = x_0$$

$$F^{\because} = F^* \quad \text{or} \quad \frac{\alpha^{\because} f(s^{\because})}{\alpha^{\because} f(s^{\because}) + f_1(s^{\because})} = \frac{\alpha^* f(s^*)}{\alpha^* f(s^*) + f_1(s^*)} \tag{12}$$

$$[b(s^*)\theta(R^+ + 0, t) + c(s^*) - b(s^{\because})\theta^* - c(s^{\because})](\frac{dR^+}{dt} + V(t)) = \lambda^+(s^*)\cdot\frac{\partial\theta(R^+ + 0, t)}{\partial x}$$

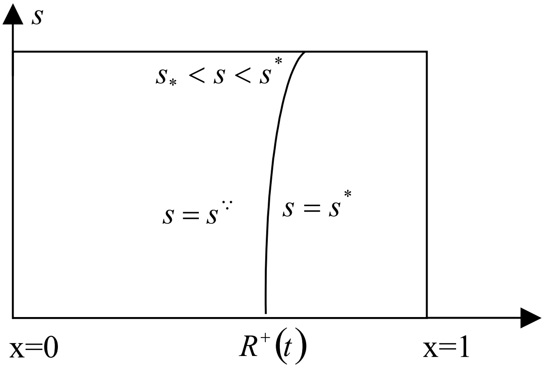

Fig. 3. Process in border R^+

Where $s^{\because} \in \{(s_*, s^*) : x = R^+ - 0\}$. The iterative scheme for a finding R^+ similarly leaves. The scheme of a choice of parameters and conditions of process is shown in Figure 3.

When $\theta = \theta^*$:

$$\frac{\partial}{\partial t}\{b(s)\cdot\theta^* + c(s)\} = \frac{\partial}{\partial x}\left(\lambda(s)\cdot\frac{\partial\theta^*}{\partial x} - (b(s)\cdot\theta^* + c(s))V(t)\right)$$

Further

$$\{b_s'(s)\cdot\theta^* + c_s'(s)\}\frac{\partial s}{\partial t} + V(t)(b_s'(s)\cdot\theta^* + c_s'(s))\frac{\partial s}{\partial x} = 0 \text{ or at } Q \equiv b(s)\cdot\theta^* + c(s)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(Q\cdot V(t)\right) = 0 \tag{13}$$

Considering the equations for a saturation $\frac{\partial s}{\partial t} + \frac{\partial}{\partial x}\left(F(s)\cdot V(t)\right) = 0$, we shall combine it with (13), then

$$\frac{\partial(Q+s)}{\partial t} + \frac{\partial}{\partial x}\left((Q+F)\cdot V(t)\right) = 0 \tag{13'}$$

Whence $(Q_s' + 1)\frac{\partial s}{\partial t} + \left((Q_s' + F_s')\cdot V(t)\right)\frac{\partial s}{\partial x} = 0$, at $b_s'(s)\cdot\theta^* + c_s'(s) + 1 \neq 0$ we shall receive

$$\frac{\partial s}{\partial t} + V(t)\{\frac{Q_s' + F_s'}{Q_s' + 1}\}\frac{\partial s}{\partial x} = 0 \text{ and } v + v_1 = V(t),\ s + s_1 = 1$$

$$s^{***} \in (0, <s>) \Rightarrow \overset{*}{X}(s,\bar{\tau}) = \bar{\tau}*\{\frac{Q'_s + F'_s}{Q'_s + 1}\} + \overset{*}{X}{}^{0}, \quad \bar{\tau} = \int_{t^*}^{t} V(\xi)d\xi$$

$$s^{***} \notin (0, <s>) \Rightarrow s = <s>, \forall x \in (R^-, R^+);$$

$$s\big|_{t=t^*} = <s>, \quad s(R^- + 0, t) = s^{\therefore}, \quad s(R^+ - 0, t) = s^{\because} \tag{14}$$

$$\frac{d}{ds}\left[\frac{Q'_s + F'_s}{Q'_s + 1}\right] = 0 \Rightarrow \Phi(s^{***}) = 0 \tag{15}$$

Also here it is necessary to consider internal kinematic conditions for the equation (13').

If $b'_s(s)\cdot\theta^* + c'_s(s) + 1 = 0$, then $\left((F'_s - 1)\cdot V(t)\right)\frac{\partial s}{\partial x} = 0$ or from (13) $\theta^* = -c'_s(s)/b'_s(s)$, that means at a constancy of properties of a liquid and as a first approximation $\theta^* \approx \dfrac{\rho_{oil}\beta_{oil} - \rho_w\beta_w}{\rho_w c_w - \rho_{oil}c_{oil}}$, where c_{oil} also c_w specific thermal capacities, and β_{oil} and β_w specific heat of fusion of oil and water (ice) accordingly. Last parity specifies, on validity of the accepted reasoning's as the temperature $\theta = \theta^*$ first of all is defined by physical properties of fluids. With corresponding conditions (9)–(10), (11)–(12) and (13)–(14) it is possible to solve the received equations known numerical methods [3, 4].

Thus, non isothermal process of a filtration (heating or cooling of a water phase) allows to operate structure of distribution of a saturation of considered area and its value effectively are numerically.

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