Tomasz AMBROZIAK*, Konrad LEWCZUK*

A METHOD FOR SCHEDULING THE GOODS RECEIVING PROCESS IN WAREHOUSE FACILITIES

Abstract: The paper discusses the role of goods receiving/shipping scheduling in the warehousing process design. A formal notation of schedules is proposed and implemented in a software application for a specific analytical example.

Keywords: warehousing, warehouse process, scheduling, warehouse designing.

1. Logistics task schedule as the determinant of a warehousing process

Warehousing process is a sequence of operations token on goods. Operations must be carried out in the correct order, as the completion of one operation (one or more times) is a precondition for the commencement of another operation. A tool for organizing work in warehouse facilities by allocating material flow operations in time, accounting for their sequence, is a *transport and warehouse process schedule*.

Schedule describes time sequence of process operations and the means of their performance. In technological terms, a schedule determines the starting and the ending moments of operations carried out in series or in series/in parallel by the relevant equipment; thus, it answers the questions: what, when, and with what equipment. A schedule describes the process in terms of both the sequence of operations and their position on the time axis.

Scheduling of operations in a warehousing process is the key element of any efficient control of material flow in warehouse facilities. A schedule may constitute a process control parameter or the process outcome or a criterion for assessment of warehouse system functioning.

* Warsaw University of Technology, Warsaw, Poland

The purpose of this paper is to determine the connections between external schedules (i.e. the delivery schedule and the shipment schedule) and the internal schedule of a warehousing process. In the context of a logistics system, scheduling is closely related to the concept of inter-operational buffering.

Scheduling of operations in a warehouse and transport process is aimed at determining the time required for the performance of such operations. This time will be referred to as the disposable time. *Net disposable time* plus the time required for technical maintenance of means of conveyance, changes between work areas and other auxiliary operations, is the *gross disposable time* for the given piece of equipment (or labour category). Gross disposable time can be read directly from a schedule of operations.

The number of transportation devices required to handle material flow is determined by the decomposition of the labour–demand of warehousing process operations within the net disposable time 3, (a similar formula can be derived for the total number of employees for subsequent categories of skilled labour):

$$n_k^U = \frac{\sum\limits_{m \in M} \lambda_{Um}^{Dk} \cdot t_{Um}^k}{t_z^k \cdot \varphi_t} \tag{1}$$

where n_k^U – number of k type devices performing operation $m \in M$.

Note the denominator of the formula (1), which is the gross disposable time for the given type of equipment times the equipment time utilization ratio. It is assumed that the total number of operations m performed daily by the relevant piece of equipment λ_{Um}^{Dk}, is determined by the logistics task, whereas the time of performance of a single operation m, t_{Um}^k, and the equipment time utilization ratio φt, are determined by the work technology and organization.

Hence, the only quantity that can be controlled is the equipment gross disposable time. If this time is increased, the fraction denominator increases accordingly; as a result, the number of devices required for the performance of the given operation is decreased, which affects the related operating costs. However, the warehouse equipment usually carries out a number of different operations. Hence, increasing the gross disposable time for one operation reduces the gross disposable time for another operation. Therefore, the equipment disposable time shall be allocated to operations in such a manner as to minimise the equipment downtime and maximise the equipment operation ratio.

Work schedules at the warehouse input and output affect the warehouse internal schedule by determining the boundary conditions for the warehousing process. This process is carried out over time. Hence, in order to meet the external schedules, the internal schedule should assure that the relevant operations are performed on time, while optimising the use of the available equipment (Fig. 1).

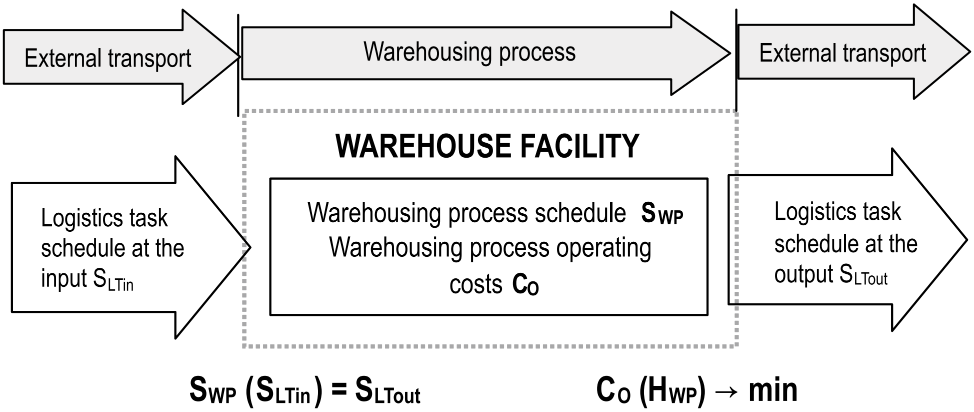

Fig. 1. *Relation between the logistics task schedules and the warehousing process schedule*

2. A method for determining the goods receiving/shipping schedule for a warehousing process under design

The presented method provides formal notation of logistics task data and formulation of constraints for the schedule at the warehouse input/output. It also enables the schedule optimisation with respect to an objective function defined in a software application on a case by case basis.

We assume that the tasks for which the goods receiving/shipping schedules are to be determined are numbered. Let I be the *set of tasks* numbered with the variable i, such as: $i \in I$, $I = \{1, 2, \dots, i, \dots, I\}$. The elements of the set I are interpreted as tasks, i.e. vehicles handled (unloaded or loaded) during the disposable time of a loading dock. Each task i is defined by the type and number of warehouse/cargo units. The type of units (e.g. palletised load units, boxes) determines the handling technology, while their number determines the time of performance of the task i using the technology j.

Let J be the *set of task performance technologies* numbered with the variable j, such as: $j \in J, J = \{1, 2, \dots, j, \dots, J\}$. The elements of the set J represent all the available technologies of task performance. It is further assumed that each task i has assigned a set J^i of technologies which can be used to perform it and $J^i \subset J$. Let Δt^i_j be the time of performance of the task i using the technology j. The set of tasks I and the set of technologies J provide the data for the matrix of handling technologies $\Delta = \left[\Delta t^i_j\right]_{I \times J}$. Each element of the set J is defined by the type of means and the number of means or the set of means involved. It is assumed that individual technologies are multipliable, i.e. a specific number of the same type may be applied. The key determinant of the elements of the technology set is that any technology can be used in one place only (loading dock) at any time.

Let $D = \{1, 2, \ldots, d, \ldots, D\}, d \in D$, be the *set of task performance places*. The service places are interpreted as loading docks in a warehouse facility. Tasks may be carried out in docks in parallel, depending on the disposable time for each dock d.

The *set of time instants*, $T = \{1, 2, \ldots, t, \ldots, T\}, t \in T$, determines the time frame for task performance and slices (quantifies) the service time. It is assumed that all time periods indicated in the analysed problem are multiples of the unit interval adopted. The task performance always starts at the beginning of the first interval, that is at the initial time instant t' and continues without interruptions until the final time instant $(t' + \Delta t)$.

Let T_d be the disposable time of the dock d determined by the initial instant, $t'_d \in T$, at which the service starts at the dock d. Let T_j be the disposable time of cargo handling technology j determined by the initial instant of the use thereof, $t'_j \in T$. These disposable times must meet the following constraints: $t'_d + T_d \leqslant T$ and $t'_j + T_j \leqslant T$.

Let us consider binary *decision variables* interpreted as allocation of the technology j from the set J^i to each task i performed at the dock d at the instant t. Let $x_j^{i,d,t}$ be interpreted as follows:

$$x_j^{i,d,t} = \begin{cases} 1, \text{ if the task } i \text{ performed at the dock } d \text{ at the instant } t \text{ has been} \\ \quad \text{assigned the material handling technology } j \\ \\ 0, \text{ otherwise} \end{cases}$$

The decision variables make up the matrix $X = \left[x_j^{i,d,t}\right]_{I \times J \times D \times T}$.

Optimisation problem *assumptions*:

- tasks must be performed without interruptions,
- each task must be performed,
- each task may be performed at any loading dock,
- the task i may be performed by the technologies j from the set J^i only,
- not all technologies j must be available at each dock d,
- the sequence of task performance is not imposed,
- the average vehicle docking and undocking times are included in the time of the task performance Δt_j^i.

Optimisation problem *constraints*:

- *Task performance continuity*: Each task i, the performance of which starts at the loading dock d using the technology j, should be performed in whole, i.e. should be continued throughout the period Δt_j^i from the chosen time instant t. It means that any subsequent decision variables that describe the status of the task i at subsequent time instants, $x_j^{i,d,t}$, $x_j^{i,d,t+1}$, $x_j^{i,d,t+2}$, $x_j^{i,d,t+\ldots}$, $x_j^{i,d,t+\Delta t_j^i}$, should be equal to 1. As the product of a sequence of binary numbers is 1 only if all the factors are 1, the operation continuity requirement is met, provided that the product of the aforementioned decision variables is 1, i.e.:

$$\prod_{s=t}^{t+\Delta t_j^i} x_j^{i,d,s} \leqslant 1 \tag{2}$$

By using a product of decision variables instead of their sum, it is possible to avoid constraining the sum to assure the task performance continuity. Owing to an assumption that each task must be performed, only one of the products (2) is equal to 1:

$$\sum_{j \in J} \sum_{d \in D} \sum_{t \in T} \left(\prod_{s=t}^{t+\Delta t_j^i} x_j^{i,d,s} \right) = 1 \qquad \forall i \in I \tag{3}$$

– *Use of loading docks*: One task at the most may be performed at the given dock d at any instant t:

$$\sum_{i \in I} \sum_{j \in J} x_j^{i,d,t} \leqslant 1 \qquad \forall d \in D \;\; \forall t \in T_d \tag{4}$$

– *Use of technologies*: The given technology j may be used to perform one task at the most at any instant t, which is expressed as the following constraint:

$$\sum_{i \in I} \sum_{d \in D} x_j^{i,d,t} \leqslant 1 \qquad \forall j \in J \;\; \forall t \in T_j \tag{5}$$

Objective function, F, of the optimisation problem depends on the needs related to the warehousing process which is subject of the input/output scheduling. In order to evaluate solutions, each feasible solution should be assigned a weight, and the general condition for the objective function optimisation (i.e. minimisation or maximisation) should be defined.

For the purpose of illustration, we assume that the best solution is the one that assures the quickest performance of all tasks. Hence, the function F will assign the highest weights to the performance of tasks which end earliest.

The maximum possible weight is T, and it is assigned if the task performance starting time is equal to the initial instant from the set T ($t = 1$) and to the task performance ending time (i.e. the service takes one time unit; see Fig. 2). Therefore, the objective function is as follows:

$$F(X) = \sum_{i \in I} \sum_{j \in J} \sum_{d \in D} \sum_{t \in T} \left(T - t - \Delta t_j^i + 2\right) \cdot \left(\prod_{s=t}^{t+\Delta t_j^i} x_j^{i,d,s} \right) \to \max_{\left\{ \left[x_j^{i,d,t} \right] \right\}} \tag{6}$$

where $\left(T - t - \Delta t_j^i + 2\right)$ – weight assigned to any feasible solution.

3. Application of the method for determining a schedule of goods receipt into a warehouse

The method will be illustrated by an analytical example.

There is a warehouse with $D = 5$ loading docks. Let the disposable time of operation of each dock be the same and equal to $T = 15$ time units. The warehouse has $J = 5$ lift trucks with operators (referred to as technologies) of various maximum speeds and various capacities, which determine their suitability for the task performance and the duration thereof. There are $I = 12$ vehicles (referred to as tasks) to be unloaded.

The time of performance of each task i using the technology j is given in the matrix $\boldsymbol{\Delta}$:

$\boldsymbol{\Delta} =$

I x J	*j=1*	*j=2*	*j=3*	*j=4*	*j=5*
i=1	3	2	3	3	4
i=2	3	2		3	4
i=3	3	2	3	3	4
i=4	3	2	3	3	5
i=5	3		3	3	5
i=6	3	2	3	2	5
i=7		2			5
i=8		2	4	2	5
i=9		2	4	2	5
i=10	3	2	4	2	5
i=11	3	2	4	2	5
i=12	3	2	4	2	5

In accordance with the constraints and the objective function discussed in the Section 2 above, an optimisation problem has been formulated. For the weights of feasible solutions see Figure 2.

Next, the example was implemented in the LINGO 9.0 application, which is a comprehensive tool designed to make building and solving linear, integer, nonlinear and global optimization models. LINGO provides a modelling language, a full featured environment for expressing problems, and a set of solvers.

Based on the aforementioned input data, the application generated a sequence of variables representing technology and dock assignment to each task over time (Fig. 3). The corresponding value of the objective function was $F(X) = 134$.

The resulting solution meets the constraints imposed on the problem. Hence, it may be believed that the value of the objective function $F(X) = 134$ determined by the application reflects the best sequence, technologies, place and time of task performance.

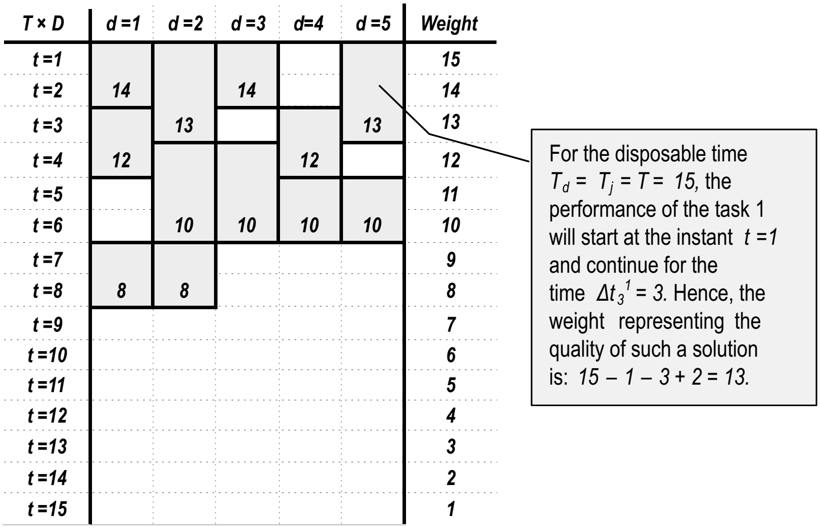

Fig. 2. *Weight assignment to tasks for the resulting schedule*

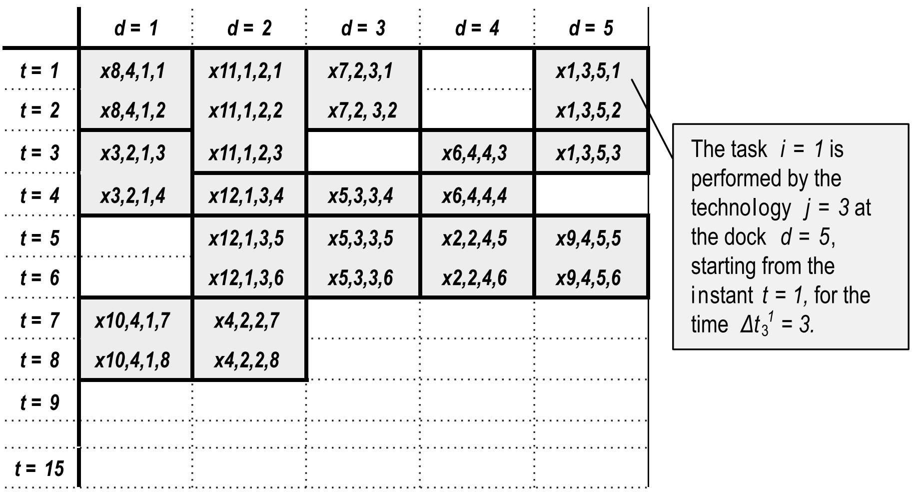

Fig. 3. *Schedule of goods receipt into a warehouse*

4. Conclusions

The method presented above is an attempt to formalise the notation of a logistics task imposed on a logistics facility in the portion related to material flow description. An external logistics task defined by the characteristics of input/output material flows related to a warehouse facility constitutes the input data for the warehousing process. The warehousing process is carried out in accordance with its internal schedule of operations.

The objective function used in the method presented may be modified depending on the warehousing process needs by changing the rules of weight assignment to solutions. In general, any change will produce a different schedule.

Implementation of the schedule of operations as an optimisation problem assures that the **definite** solution (corresponding to the objective function extreme) is found. However, this type of notation and its explicit nature have some adverse consequences.

All decision variables in the problem are binary and their total number corresponds to the product of the dimensions of the matrix of decision variables, X. In this case, this is a four-dimensional matrix. The presented version of the optimisation problem with such decision variables is NP-complete, so the problem is NP-hard. This problem feature comes from specific problem properties. The time of solving such problems is a fast ascending function of the number of variables.

Therefore, potential further research may focus on solving the aforementioned optimisation problem with methods developed for problems which involve a large number of variables, such as neural networks or fuzzy sets.

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