Frequency estimation in electric power system measurements

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Abstract: The paper investigates the performance of recently proposed frequency estimation methods in the presence of typical power system measurements disturbances, i.e. sinusoids and additive noise. The tests are conducted for the polyphase Prony method, the polyphase smart DFT, and the discrete-time frequency-gain transducer. Considered methods are capable of unbiased one-cycle frequency estimation. The robustness against disturbances is obtained by the application of the proposed prefilter. Consider methods with the initially filtered measurement signal perform significantly better than the standard PMU (phasor measurement unit) P class and M class signal processing models.

Keywords: discrete Fourier transform, spectral filtration, smart DFT, Prony method, discrete-time frequency-gain transducer, least squares solution

ESTYMACJA CZĘSTOTLIWOŚCI W POMIARACH
W SYSTEMIE ELEKTROENERGETYCZNYM

Streszczenie: W pracy przedstawiono zastosowanie nowoczesnych metod estymacji częstotliwości w przypadku obecności typowych zaburzeń pomiarowych występujących w systemie elektroenergetycznym, tj. sygnałów sinusoidalnych i szumu addytywnego. W badaniach uwzględniono polifazową metodę Prony'ego, polifazowe smart DFT oraz przetwornik częstotliwość-wzmocnienie z czasem dyskretnym. Powyższe metody umożliwiają nieobciążoną estymację częstotliwości na podstawie jednego okresu. Odporność na zakłócenia uzyskano dzięki zastosowaniu zaproponowanego filtra wstępnego. Badane metody działające na wstępnie przefiltrowanym sygnale wykazują znacznie mniejsze błędy pomiaru częstotliwości niż standardowe modele PMU (*phasor measurement unit*) w klasie P i w klasie M.

Słowa kluczowe: dyskretne przekształcenie Fouriera, filtracja częstotliwościowa, metoda Prony'ego, przetwornik częstotliwość-wzmocnienie, metoda najmniejszych kwadratów

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1. Introduction

Estimation of a fundamental frequency is a basic problem in electric power system measurements. Many frequency estimation methods were designed so far but the problem is still under intense research. The design of frequency estimation method is difficult because it must not only be accurate, but also it must be robust against disturbances and computationally feasible. The most severe disturbances in electric power system are interfering sinusoids, i.e. harmonics, interharmonics, and subharmonics. Disturbances with frequency close to the system nominal frequency, i.e. the out-of-band interference, and the 2nd harmonic, are the most difficult to deal with. Also, the immunity to additive noise should be considered, although the levels of measured voltages and currents are high, and signal to noise ratio is moderate to high. Instantaneous frequency, amplitude and phase of a measured signal are time-varying, thus an estimation algorithm should have low latency, preferably several nominal cycles or less. It should also have a fast response to amplitude and phase steps. Above requirements are contradictory in many aspects, e.g. filtering out harmonic disturbance requires long impulse response that increases system latency. Available algorithms made different trade-offs, i.e. they sacrifice one property for another, e.g. in general a shorter time estimator is expected to be more "instantaneous" by the cost of less robustness against disturbances.

Different systematics can be applied for frequency estimation methods, and still some methods can belong to more than one category. In this writing we restrict our interest to the selected methods based on: the DFT (discrete Fourier transform), a FIR (finite impulse response) filter, and the LS (least squares) solution.

The DFT based frequency estimation is biased by the spectral leakage and the picket fence effect (Harris 1978) both occurring for noncoherent sampling. The impact of both phenomena can be mitigated on the stage of data acquisition by synchronous sampling, which is done by adjusting the sampling frequency to the signal frequency by the PLL (phase locked loop) (IEC 61000-4-7:2002), or by the software coherent resampling (Borkowski and Bień 2009). For non-coherently sampled signal the spectral leakage is reduced by time windows with high attenuation of the spectral side lobes, and the picket fence effect is overcome by the Interpolated DFT (IpDFT) algorithms. Application of a time window other than the rectangular one decreases frequency estimation bias by the cost of increasing estimation variance. Closed-form IpDFT formulas for a family of $\sin^{\alpha}(x)$ windows, that are generalization of the Rife-Vincent class I windows and are also referred to as the maximum sidelobe decay windows, are given in Duda and Barczentewicz (2014). For $\alpha = 0$ and $\alpha = 2$ the $\sin^{\alpha}(x)$ window is the rectangular window and the Hann window, respectively. For arbitrary window IpDFT formula can be derived by polynomial approximation (Duda 2011). IpDFT is also used for analysis of damped sinusoidal signals (Duda et al. 2011). The IpDFT algorithm proposed in Wang et al. (2021) is free of the spectral leakage, and enables frequency and damping estimation of a single sinusoid even from an observation shorter than one signal cycle.

The Smart DFT (SmDFT) (Yang and Liu 2000) uses the ratio of three DFT bins computed with one sample shift. The SmDFT is not biased be the spectral leakage nor by the picket fence effect. In Duda and Zieliński (2021) it is shown that the downsampled (polyphase) SmDFT has higher noise immunity than the original SmDFT. Exemplary results of frequency estimation with the polyphase SmDFT are presented next.

FIR filter-based frequency estimation is used in the IEC/IEEE standard for synchrophasor measurements (IEC/IEEE 60255-118-1:2018). The phasor measurement unit (PMU) signal processing model contains quadrature demodulator working with the power system nominal frequency. The phasor is next obtained by lowpass filtration, and the frequency is computed as a time derivative of signal's phase. Applied lowpass filter determines measurement properties of the PMU, i.e., accuracy and disturbance immunity. Unfortunately, PMU with the standard M class lowpass filter is not compliant with the standard. Compliant PMU implementation can be obtained with the flat-top filters (Duda and Zieliński 2016, Duda et al. 2016).

Implementation of the Taylor–Fourier transform results in FIR filters for phasor, frequency and rate of change of frequency estimation (Platas-Garza and de la O Serna 2010, de la O Serna et al. 2021) with the advantages of negative image component and harmonics attenuation. The Taylor–Fourier transform basis contains not only phasor, as the DFT does, but also time derivatives of the dynamic phasor Taylor series expansion. The flat-top passband FIR Hilbert transformers, designed by the Taylor–Fourier transform, can also be designed with the flat-top filters (Duda et al. 2016) with the advantage of higher sidelobes attenuation (Duda et al. 2018).

FIR filter-based methods able for low latency estimation are the complex bandpass (CxBp) filters (Xu et al. 2021) and the discrete-time frequency-gain transducer (DTFGT) (Duda and Zieliński 2022a). Both, the CxBp filters and the DTFGT, can estimate frequency based on approximately one nominal cycle observation. Exemplary results of frequency estimation with the DTFGT are presented next.

The most popular frequency estimation method based on LS solution is the Prony method (Kay and Marple 1981, Zieliński and Duda 2011, Duda and Zieliński 2013). The original Prony method is sensitive to noise, however by introducing its polyphase implementation the noise immunity of the Prony method is significantly improved (Duda and Zieliński 2022b). Exemplary results of frequency estimation with the polyphase Prony method are presented next.

In this paper we investigate the properties of three low-latency unbiased frequency estimators namely: the polyphase SmDFT (Duda and Zieliński 2021), the DTFGT (Duda and Zieliński 2022a) and the polyphase Prony method (Duda and Zieliński 2022b). All above estimators work with approximately one nominal frequency period. The considered estimators are very close to the optimal one for sinusoidal signal disturbed only by additive noise. For sinusoidal interference their performance is improved by signal enhancing with a prefilter. As the harmonic interference is a very common disturbance, we propose the FIR prefilter with the impulse response being a self-convolution of the rectangular window, with length equal to the nominal period, and modulated to the system nominal frequency. This prefilter has zeros in frequency magnitude response at nominal harmonic frequencies. For reference the results are compared with the standard PMU signal processing models in P class and M class (IEC/IEEE 60255-118-1:2018).

The paper is organized as follows. Section 2 defines the signal model. Section 3 describes investigated frequency estimation algorithms. Section 4 presents proposed prefilter for signal enhancing before frequency estimation. The results of frequency estimation for signal without disturbances, with additive noise and with sinusoidal interference are presented and discussed in Section 4. Section 5 concludes the paper.

2. Signal model

A continuous-time voltage or current signal in AC electric power grid is modelled by (IEC/IEEE 60255-118-1:2018):

$$x(t) = X_m(t)\cos\left[\Theta(t)\right] + D(t) \tag{1}$$

where: t is time in seconds, X_m is the peak magnitude of sinusoidal AC signal, $\Theta(t)$ is the angular position of the sinusoidal AC signal in radians, and $D(t)$ is a disturbance signal that contains additive contributions to the signal, including, but not limited to harmonics, noise, DC offset and out-of-band interference.

The angular position is the sum of the phase due to the nominal frequency f_0 in hertz, and instantaneous phase $\phi(t)$:

$$\Theta(t) = 2\pi f_0 t + \phi(t) \tag{2}$$

The complex value dynamic phasor of the signal (1) is given by:

$$X(t) = \frac{X_m(t)}{\sqrt{2}} e^{j\phi(t)} \tag{3}$$

Instantaneous frequency of the signal (1) is a time derivative of the cosine function argument:

$$f(t) = \frac{d\Theta(t)}{dt} = f_0 + \frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{4}$$

E.g. if the instantaneous phase is a linear function of time $\phi(t) = 2\pi\Delta f t + \phi_0$, then the instantaneous frequency deviates from the nominal frequency by Δf [Hz], i.e. $f(t) = f_0 + \Delta f$.

The parameters of the measurement signal (1), i.e., instantaneous frequency, amplitude and phase, are estimated based on its discrete-time representation obtained by periodic sampling with sampling frequency f_s:

$$x[n] = X_m[n]\cos\left(2\pi\frac{f_0}{f_s}n + \phi[n]\right) + D[n], \quad n = 0, 1, \ldots, N-1 \tag{5}$$

where N is the number of signal samples. Nominal frequency ω_0 in radians, also referred to as pulsation or angular pulsation, is given by:

$$\omega_0 = 2\pi \frac{f_0}{f_s} = \frac{2\pi}{N_0}, \quad 0 < \omega_0 < \pi \tag{6}$$

where $N_0 = f_s/f_0$ is the number of samples in nominal period.

3. Considered frequency estimation methods

3.1. Polyphase Prony method

The polyphase Prony method was introduced for a single undamped sinusoidal signal in Duda and Zieliński (2021), and extended for a multifrequency damped sinusoidal signal in Duda and Zieliński (2022b). In comparison to the original Prony method the polyphase Prony method has significantly higher noise immunity. It exploits the LS solution of a set of equations build on autoregressive difference equation for sinusoidal signal. For an undamped single sinusoid defined as:

$$y[n] = A_0 \cos(\omega_0 n + \varphi_0), \; n = 0, 1, 2, \ldots, N-1 \tag{7}$$

the current signal value $y[n]$ is determined by two previous values $y[n-L]$ and $y[n-2L]$:

$$y[n] = cy[n-L] - y[n-2L], \; n = 2L, \; 2L+1, \ldots, N-1 \tag{8}$$

where $c = 2\cos(\omega_0 L)$, and $L = 1, 2, ...,$ is a downsampling factor. It is shown in Duda and Zieliński (2021, 2022b) that the highest noise immunity is obtained for L selected such as to obtain 4 samples per signal cycle. For $L = 1$ we get the original Prony method. The frequency is computed as:

$$f[n] = \frac{f_s}{2\pi L} \cos^{-1}\left(\frac{c}{2}\right) \tag{9}$$

where the coefficient c is obtained by LS solution:

$$c = \left(\mathbf{E}^T \mathbf{E}\right)^{-1} \mathbf{E}^T \mathbf{b} \tag{10}$$

where the vectors $\mathbf{E} = \begin{bmatrix} x_L \\ \vdots \\ x_{N-1-L} \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} x_0 + x_{2L} \\ \vdots \\ x_{N-1-2L} + x_{N-1} \end{bmatrix}$ are filled with the samples of the measured signal (5).

3.2. Polyphase Smart DFT

The SmDFT was introduced in Yang and Liu (2000). In Duda and Zieliński (2021) it was shown that by introducing downsampling factor L into the solution the noise immunity is significantly improved. The polyphase SmDFT is defined based on the autoregressive difference equation (8) and linearity of the DFT. The frequency is computed as:

$$f[n] = \frac{f_s}{2\pi L} \cos^{-1}\left(\frac{DFT_{k=1}\{b\}}{2DFT_{k=1}\{E\}} \right) \tag{11}$$

where L, $\mathbf{E}$, and $\mathbf{b}$ are the same as previously, and $\mathrm{DFT}_{k=1}\{\cdot\}$ stands for the computation of the DFT bin with index $k = 1$ from the signal within the brackets, thus it is assumed that the observed signal spans approximately one signal cycle.

3.3. Discrete-time frequency-gain transducer

The DTGFT proposed in Duda and Zieliński (2022a) uses a FIR filter with a slope in frequency response. The frequency is computed based on the gain G of this filter:

$$f[n] = \frac{f_s}{2\pi L} \cos^{-1}(2G - 1) \tag{12}$$

where the gain is computed as:

$$G = \sqrt{\frac{h_{LP}[n] * \left(h_0^L[n] * x[n]\right)^2}{h_{LP}[n] * (x[n])^2}} \tag{13}$$

where the asterisk * denotes a linear convolution. According to (13) measured signal $x[n]$ is firstly filtered by the slope filter $h_0^L[n]$:

$$h_0^L[n] = \begin{cases} h_0\left[\dfrac{n}{L}\right], & n = 0, \pm L, \pm 2L \\ 0, & \textit{otherwise} \end{cases} \tag{14}$$

where $h_0[n] = [1/4, 1/2, 1/4]$, and next the filtered signal is squared and filtered by the lowpass filter $h_{LP}[n]$:

$$h_{LP}[n] = 1, \quad n = 0, \ldots, N_{LP} - 1 \tag{15}$$

3.4. Standard PMU signal processing model

The standard PMU signal processing model (IEC/IEEE 60255-118-1:2018) exploits quadrature demodulator followed by the lowpass filter. In the P class a FIR filter with triangular impulse response is used, i.e., the convolution of two rectangular pulses each with the width equal to nominal cycle. This lowpass filter completely attenuates harmonics for nominal frequency. In the M class a FIR filter is designed by the "brick wall" method, also known as the window method (Oppenheim et al. 1999), with the Hamming window. The phasor is computed as the result of lowpass filtration after quadrature demodulation:

$$X[n]=\left(x[n]e^{-j\omega_0 n}\right)*h_{LP}[n] \tag{16}$$

where $h_{LP}[n]$ is an applied, P class or M class, lowpass filter. The frequency is next computed by approximation of time derivative:

$$f[n]=f_0+\frac{f_s}{2\pi}\frac{\phi[n+1]-\phi[n-1]}{2} \tag{17}$$

where $\phi[n]$ = angle($X[n]$).

4. Prefilter for signal enhancement

The prefilter is used for the attenuation of the additive disturbance $D[n]$ in the measurement signal $x[n]$ (5). In the standard a lowpass filter is used for this purpose in the P class and in the M class PMUs. Frequency estimation methods under research, i.e. the polyphase Prony, the polyphase SmDFT, and the DTGFT do not apply quadrature demodulation, thus the prefilter is a bandpass filter around nominal frequency. For an even value of N_0 the prototype prefilter impulse response is defined as a rectangular pulse with the width equal to the period of nominal frequency ω_0, modulated by a cosine with the nominal frequency:

$$h_0[n]=\frac{2}{N_0}\cos\left(\omega_0\left(n-\frac{N_0}{2}+0,5\right)\right), \quad n=0,\ldots,N_0-1 \tag{18}$$

The self-convolved prefilter of order P is then obtained by linear convolution of the prefilter of order $P-1$ and the prototype prefilter:

$$h_P[n]=h_{P-1}[n]*h_0[n], \quad P=1,2,\ldots \tag{19}$$

Figure 1 depicts impulse responses and frequency magnitude responses for the prefilters of orders $P = 0, 1, ..., 5$ for $N_0 = 16$. Due to iterated convolution (19) the length of the impulse response of the filter $h_p[n]$ elongates, in each iteration, by $N_0 - 1$ taps and equals $N = N_0 (P + 1) - P$. The legend gives the length N of impulse response of each $h_p[n]$ filter, and the length of the filter expressed in the number of nominal cycles N/N_0. It is observed that with the increase of order P the attenuation in the whole stopband, and not only at harmonic frequencies, significantly increases. Each $h_p[n]$ filter has a gain equal to 1 for the nominal 50 Hz frequency. With the frequency increase up to around 56.5 Hz the gain increases to a local maximum, e.g. the maximum gain equals to 1.03 for the $h_0[n]$ filter, and 1.22 for the $h_s[n]$ filter, this however has no effect on frequency estimation.

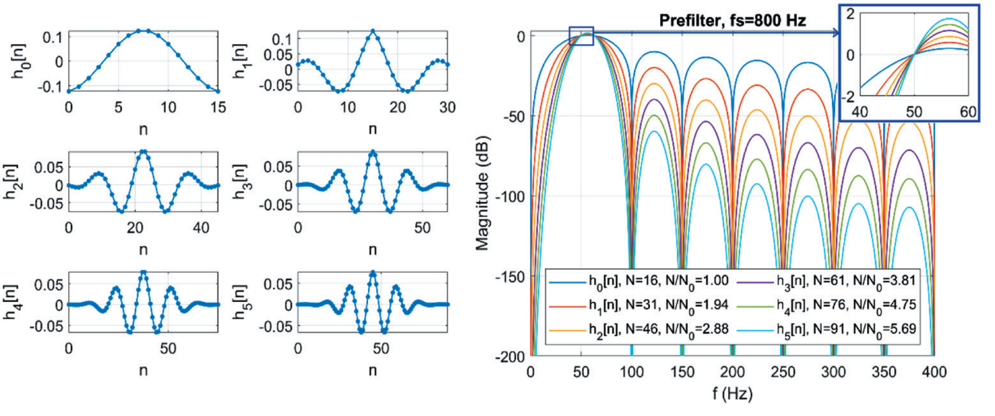

Fig. 1. Prefilter impulse responses (left) and magnitude responses (right) for orders $P = 0, 1, ..., 5$

5. Results

The following sections present results of frequency estimation for a signal without disturbance, with additive zero-mean white Gaussian noise (WGN), sinusoidal interference, and multi-disturbance composed of both WGN and interfering sinusoid. The sampling frequency $f_s = 800$ Hz, the same as in the standard reference signal processing model (IEC/IEEE 60255-118-1:2018), is used in all simulations. The investigated frequency estimators are: the polyphase Prony method (denoted by Prony in all legends), the polyphase SmDFT (denoted by SmDFT), and the DTGFT. In all methods $L = 4$ is set, as this results in 4 samples per nominal period, which in turn guarantees the highest noise immunity in these methods. For each test the results are presented for: considered frequency estimators working alone, and considered frequency estimators working with the prefilter $h_0[n]$, with comparison to the standard P class PMU model

(IEC/IEEE 60255-118-1:2018), and working with the prefilter $h_7[n]$, with comparison to the standard M class PMU model (IEC/IEEE 60255-118-1:2018). The investigated estimators have the length of 1, 2, and 8.5 nominal cycles, the P class PMU model is 2 cycles estimator, and M class PMU model is 8.9 cycles estimator. The results are presented in terms of frequency error (FE) in hertz defined in (IEC/IEEE 60255-118-1:2018) as the difference between the measured frequency and the reference frequency. Further, the maximum absolute FE or the root-mean-square error (RMSE) of FE are presented.

5.1. Steady state frequency test

Steady state frequency compliance test is defined in (IEC/IEEE 60255-118-1:2018). In the P class PMU the range of fundamental frequency is 48–52 Hz, and in the M class PMU it is 45–55 Hz. In both classes the maximum absolute frequency error must not exceed 0.005 Hz. Obtained results are presented in Figure 2. It is observed that the methods under investigation, with and without the prefilter, are practically unbiased. The P class PMU and the M class PMU both are not compliant with its own standard (IEC/IEEE 60255-118-1:2018), as the error exceeds imposed limits.

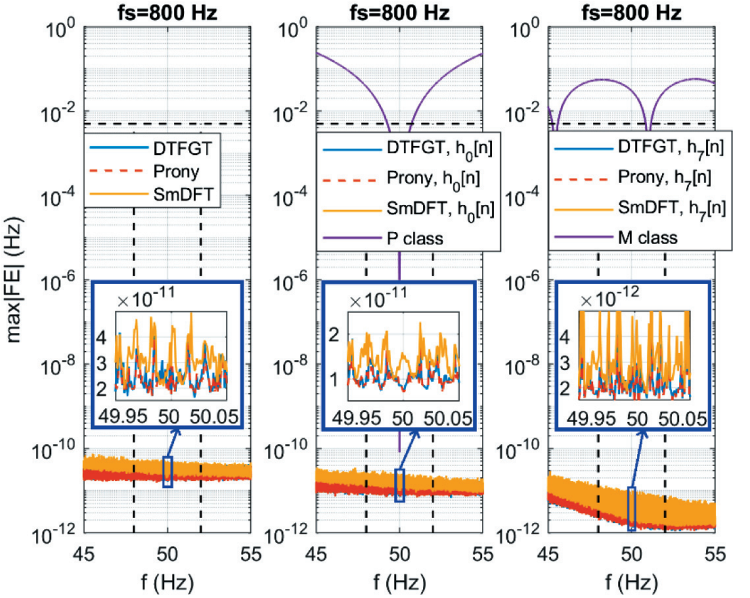

Fig. 2. Maximum absolute FE as a function of fundamental frequency: frequency estimators alone – 1 cycle estimation (left), frequency estimators with the prefilter $h_0[n]$ and the standard P class estimator – 2 cycles estimation (middle), and frequency estimators with the prefilter $h_7[n]$ – 8.5 cycles estimation and the standard M class estimator – 8.9 cycles estimation (right). Black dotted lines show the standard limits in the P class and the M class. Errors for the DTFGT (blue) are overlapped by the Prony method (dashed red)

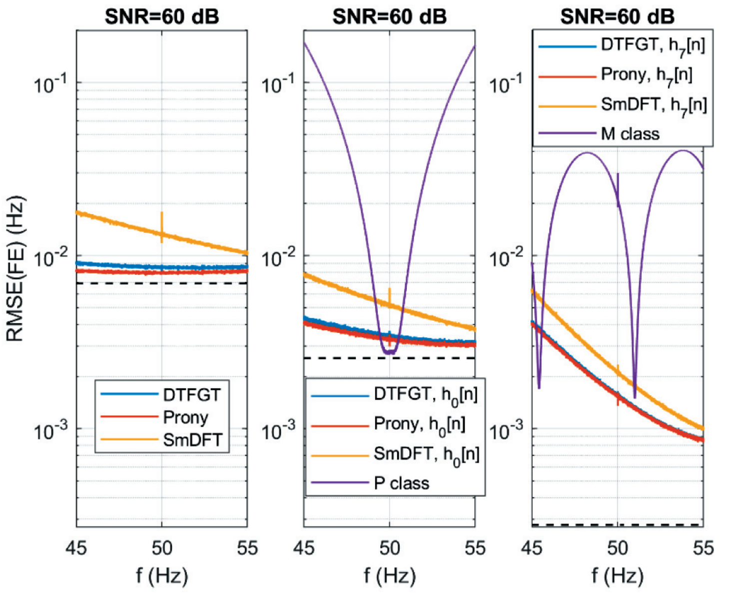

Fig. 3. RMSE of FE as a function of fundamental frequency for a signal with 60 dB SNR. Denotations as in Figure 2. Black dotted lines show the CRLB

5.2. Noise

Noise propagation through a frequency estimation algorithm is a fundamental property. Statistically efficient estimator is unbiased and its variance is determined by the Cramér–Rao lower bound (CRLB) (Kay 1993):

$$\operatorname{var}(\omega) = \frac{12}{N\left(N^2 - 1\right)SNR}, \quad SNR = \frac{X_m^2}{2\sigma^2} \tag{20}$$

where ω is in radians as in (5) and (6), *SNR* stands for the signal-to-noise ratio and σ^2 is a variance of additive WGN. The variance of frequency estimation (20) is inversely proportional to the third power of the number of samples N thus noise immunity increases very fast with the number of samples. Figure 3 presents the RMSE of FE in comparison to the CRLB expressed as the standard deviation in hertz. It is observed that without the prefilter the DTFGT and the polyphase Prony are close to optimal frequency estimator, as they closely track the CRLB, however application of the prefilter deteriorates this optimality, and the longer the prefilter is the further the results are from the CRLB. The P class PMU is close to the CRLB only in a very narrow band of approximately 49.6–50.4 Hz, and the M class PMU is generally sensitive to additive noise.

5.3. Sinusoidal interference

In the standard (IEC/IEEE 60255-118-1:2018) sinusoidal interference is separately treated in the harmonic distortion test and the out-of-band interference test. In this study the frequency of interfering sinusoid is swept, with the step of 0.1 Hz, in the range from 75 to 400 Hz (i.e. the half of the sampling rate), what includes out-of-band interference and harmonics, but also interharmonics. Figure 4 depicts maximum absolute FE as a function of a 10% interfering sinusoid frequency for a signal with nominal 50 Hz frequency. From the investigated methods without a prefilter only the polyphase SmDFT rejects even harmonics except the 2nd one. Application of the rectangular prefilter $h_0[n]$ benefits in an ideal rejection of all nominal harmonics. By increasing the order of the prefilter also interharmonics are strongly attenuated. The P class PMU performs better than the polyphase SmDFT with $h_0[n]$ prefilter, but slightly worse than the DTFGT, and the polyphase Prony with $h_0[n]$ prefilter, and the M class PMU is the clear outsider.

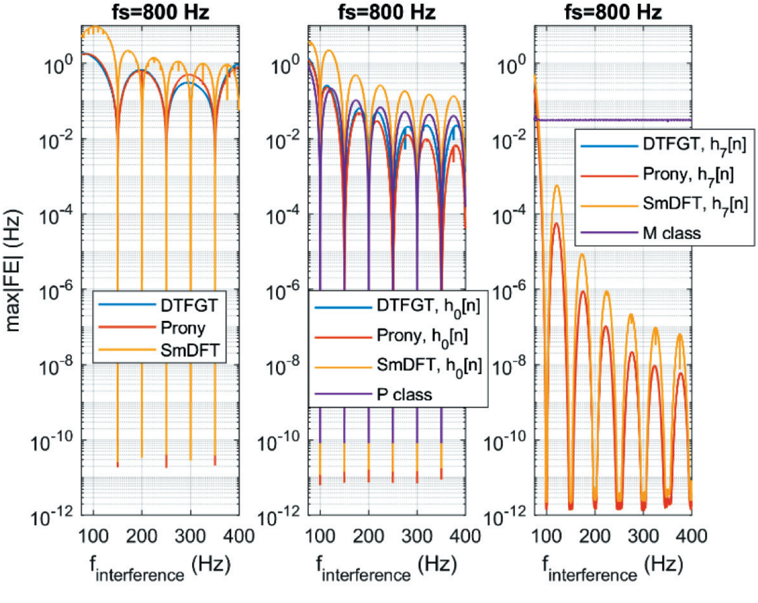

Fig. 4. Maximum absolute FE as a function of a 10% interfering sinusoid frequency for a signal with nominal 50 Hz frequency. Denotations as in Figure 2

5.4. Multi-disturbance

In practice the measurement signal is simultaneously affected by different disturbances. A frequency estimation algorithm often compromises sensitivity to some disturbances to obtain robustness against other disturbances.

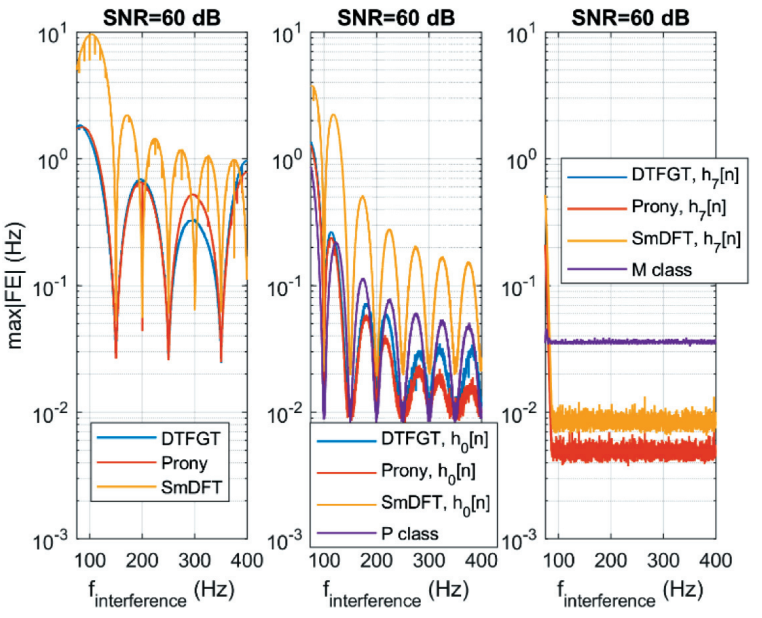

Fig. 5. Maximum absolute FE as a function of interfering sinusoid frequency for signal with 60 dB SNR. Denotations as in Figure 2. Errors for the DTFGT (blue) are overlapped by the Prony method (red)

Figure 5 depicts maximum absolute FE for a signal simultaneously disturbed by interfering sinusoid and additive WGN. It is observed that the DTFGT and the polyphase Prony method are the best performers in this test. The polyphase SmDFT without the prefilter and with $h_0[n]$ prefilter performs worse. The M class PMU has the highest error except the range from 75 to 83 Hz where it has the smallest error on the level of 0.04 Hz.

6. Conclusions

The paper presents the performance of the following short-cycle frequency estimators: the polyphase Prony method, the polyphase SmDFT, and the DTGFT, in the presence of typical electric power system disturbances, i.e. a sinusoid and an additive noise. It is shown that these frequency estimators outperform PMU signal processing models defined in IEC/IEEE 60255-118-1:2018 and the robustness against disturbances in frequency estimation methods can be obtained by enhancing the signal with a prefilter. Thus the accuracy can be increased by the cost of higher latency, with the advantage that only signal preprocessing stage is changed and frequency estimation part remains the same. IEC/IEEE 60255-118-1:2018 defines also dynamic testing not considered in this paper, but it is known from Duda and Zieliński (2022a), that e.g. the DTFGT can be used for fully compliant PMU implementation.

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