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INTEGRATED SCHEDULING IN A SUPPLY CHAIN BY MIXED INTEGER PROGRAMMING

Abstract: A mixed integer programming approach is proposed for a long-term, integrated scheduling of material manufacturing, material supply and product assembly in a customer driven supply chain. The supply chain consists of three distinct stages: manufacturer/supplier of product-specific materials (parts), producer where finished products are assembled according to customer orders and a set of customers which generates final demand for the products. The overall problem is how to coordinate manufacturing and supply of parts and assembly of products so that the total supply chain inventory holding cost and the production line start-up and parts shipping costs are minimised. Numerical examples are presented and some computational results are reported.

Keywords: scheduling, mixed integer programming, supply chain.

1. Introduction

In a customer driven supply chain, the procurement policies depend on the production schedule that is driven by customer orders for finished products. Coordination of raw material manufacturing and supply, and finished goods production and distribution is one of the main issues of supply chain management, e.g., Erenguc *et al.* (1999), Kolisch (2000), Chen and Vairaktarakis (2005), Kaczmarczyk *et al.* (2006). A joint schedule for manufacturing and supply of parts and for assembly of finished products should aim at reaching a high customer service level and the best tradeoff between total supply chain inventory holding cost and both the producer's and the supplier's set-up and shipping costs.

In this paper a mixed integer programming approach is proposed for the multi-objective, integrated scheduling of part manufacturing, part supply and product assembly in a customer driven supply chain. The supply chain consists of the supplier stage made up of identical production lines in parallel; the producer stage which is a flexible assembly line made up of several assembly stages in series; and a set of customers. The overall problem is how to coordinate manufacturing and supply of

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parts and assembly of products with respect to limited capacities and required customer service level to minimise total inventory holding cost. In a companion paper (Sawik, 2007c), the maximum level of the total inventory is minimised. Monolithic vs. hierarchical approach for multi-objective integrated scheduling in a supply chain is proposed in Sawik (2008a, 2008b), where the supply chain consists of a single (Sawik, 2008a) or multiple (Sawik, 2008b) manufacturers (suppliers) of parts. In Sawik, 2008b, the integrated scheduling of manufacturing and delivery of parts and assembly of products is combined with supplier selection for each customer order and due date setting for some orders.

In the literature, on production planning and scheduling the integer programming models have been widely used, e.g. Pochet and Wolsey (2006), Sawik (2007a, 2007b). For example, a mixed integer programming formulation for the integrated assembly scheduling and fabrication lot sizing in make-to order production is proposed in Kolisch (2000), with the objective function that minimises the total inventory holding and fabrication setup cost.

The paper is organised as follows. Description of the integrated scheduling in a customer driven supply chain is provided in Section 2. The mixed integer programming formulation is presented in Section 3. Numerical examples and some computational results are reported in Section 4, and conclusions are made in the last section.

2. Problem description

The supply chain consists of three distinct stages: manufacturer/supplier of product-specific materials (custom parts), producer where finished products are assembled according to customer orders and a set of customers which generates final demand for the products.

At the manufacturing stage, product-specific parts are manufactured for all product types that are assembled at the producer stage. The manufacturing stage is made up of m_M identical production lines in parallel. Let K be the set of product-specific part types and q_k processing time of part type $k \in K$. The planning horizon consists of t_h planning periods (e.g., business days) of equal length l (e.g., expressed in hours or minutes) and let $T = \{1, \dots t_h\}$ be the set of planning periods. In each period, at most one part type can be manufactured on each production line. When a production line switches from one part type to another, a start-up time should be considered at the beginning of the period. The start-up times are sequence-independent and are assumed to be equal for all part types. Let σ be the start-up time of each production line.

The manufactured parts are next transported to the producer at most once per period. The size of each shipment is limited by the minimum and the maximum allowed shipping lot: $\underline{V}$ and $\overline{V}$, respectively. The transportation time is assumed to be constant and equal to one period for every shipping lot. Parts manufactured in period t can be transported to the producer stage in the same period and can be used for products assembly in period $t + 1$, at the earliest.

The producer stage is a flexible assembly line made up of m_A assembly stages in series and is capable of assembling various types of products in a make-to-order environment responding directly to customer orders. Let J be the set of customer orders known ahead of time, K - the set of product types (identical to the set of product-specific part types) and J_k the subset of orders for product type $k \in K$ (i.e. requiring part type k to be assembled). Each order $j \in J$ is described by a triple (r_j, d_j, o_j), where r_j is the order ready date (e.g., the earliest period of material availability), d_j is the customer requested due date (e.g., customer required shipping date) and o_j is the size of order (quantity of ordered products of a specific type).

Each order requires processing at various assembly stages, but it must be completed in a single planning period (see Sawik 2007a). Let $p_{ij} \geqslant 0$ be the processing time in stage $i \in A$ of each product in order $j \in J$ and c_{it} – the total processing time available at stage $i \in A$ in period $t \in T$. The problem objective is to find an integrated schedule for manufacturing and supply of parts and assembly of products such that the total supply chain inventory holding cost and the production line start-up and part shipping costs are minimised.

3. Problem formulation

In this section a mixed integer program is presented for the integrated scheduling of products assembly and parts manufacturing and supply in the customer driven supply chain (for definitions of the decision variables, see Table 1).

3.1. Supply chain inventory

In the supply chain under study, the following three types of inventory can be distinguished:

I^1_{kt} – supplier output inventory of manufactured part type k in period t,

I^2_{kt} – producer input inventory of supplied part type k in period t, including parts transported in period t from supplier to producer,

I^3_{kt} – producer output inventory of finished product type k in period t,

$$I^1_{kt} = I^1_{k0} + \sum_{\tau=1}^{t} (u_{k\tau} - v_{k\tau});\ k \in K, t \in T \tag{1}$$

$$I^2_{kt} = I^2_{k0} + \sum_{\tau=1}^{t} (v_{k\tau} - \sum_{j \in J_k} o_j x_{j\tau});\ k \in K, t \in T \tag{2}$$

$$I^3_{kt} = \sum_{\tau=1}^{t} \sum_{j \in J_k : d_j > t} o_j x_{j\tau};\ k \in K, t \in T \tag{3}$$

where I^1_{k0}, I^2_{k0} is the beginning inventory of part type k at supplier, producer, respectively.

The total inventory can be optimised by minimising either its maximum level (see Sawik 2007c) or its holding cost H_{sum} defined below:

$$H_{sum} = \sum_{k \in K} \sum_{t \in T} (h_k^1 I_{kt}^1 + h_k^2 I_{kt}^2 + h_k^3 I_{kt}^3). \tag{4}$$

The inventory holding costs h_k^1 and h_k^2, h_k^3, of manufacturing parts stored at a supplier, as well as parts and finished products stored at a producer, respectively satisfy the inequalities $h_k^1 < h_k^2 < h_k^3; k \in K$, due to the value-added activities down through the supply chain.

3.2. Mixed integer programming formulation

In the mixed integer program presented below the objective function represents part and product inventory holding costs, production lines start up costs and part shipping costs.

The unit costs γ_w and γ_z of the shipment and start-up should take such values that the numbers of line start ups and part shipments are kept at relative minimum with respect to the inventory holding costs.

Table 1. *Decision variables*

u_{kt}	=	manufacturing lot of part type k in period t
v_{kt}	=	transportation lot of part type k in period t
w_t	=	1, if a shipment of parts is scheduled for period t, otherwise $w_t = 0$
x_{jt}	=	1, if customer order j is assigned to planning period t; otherwise $x_{jt} = 0$ (order assignment variable)
y_{kt}	=	number of parallel production lines setup for processing part type k in period t
z_{kt}	=	number of parallel production lines started up in period t to process part type k after processing another part type

Model SAMS: *Scheduling assembly, manufacturing and supply*

Minimise

$$H_{sum} + \gamma_w \sum_{t \in T} w_t + \gamma_z \sum_{k \in K} \sum_{t \in T} z_{kt} \tag{5}$$

subject to

1. Customer order non-delayed assignment constraints

- each customer order is assigned to exactly one planning period not later than its due date:

$$\sum_{t \in T:\, r_j \leqslant t \leqslant d_j} x_{jt} = 1;\ j \in J \tag{6}$$

2. Assembly capacity constraints

- in every period, the demand for capacity at each assembly stage cannot be greater than the maximum available capacity in this period;
- in period $t = 1$ the demand for each part type k cannot be greater than the initial inventory of this part type:

$$\sum_{j \in J} p_{ij} o_j x_{jt} \leqslant c_{it};\ i \in A, t \in T \tag{7}$$

$$\sum_{j \in J_k:\, r_j = 1} o_j x_{j1} \leqslant I^2_{k0};\ k \in K \tag{8}$$

3. Manufacturing line set-up and start-up constraints

- in every period total number of production lines set up for manufacturing different part types is not greater than total number m_M of available lines;
- all production lines set up for part type k in period 1 should be started up to manufacture this part type;
- in every period $t > 1$, the number of production lines started up for part type k cannot be less than the difference between the number of lines set up for this part type in periods t and $t - 1$;
- in every period $t > 1$, the number of production lines started up for part type k cannot be greater than the number of lines set up for part type k in this period and cannot be greater than the number of lines set up for the other part types or idle in period $t - 1$:

$$\sum_{k \in K} y_{kt} \leqslant m_M;\ t \in T \tag{9}$$

$$z_{k1} = y_{k1};\ k \in K \tag{10}$$

$$z_{kt} \geqslant y_{kt} - y_{k,t-1};\ k \in K, t \in T : t > 1 \tag{11}$$

$$z_{kt} \leqslant y_{kt};\ k \in K, t \in T : t > 1 \tag{12}$$

$$z_{kt} \leqslant m_M - y_{k,t-1};\ k \in K, t \in T : t > 1 \tag{13}$$

4. Manufacturing capacity constraints

- in every period t, the production volume of part type k cannot be greater than the maximum volume corresponding to the capacity assigned to part type k in this period:

$$u_{kt} \leqslant \lfloor (l - \sigma)/q_k \rfloor z_{kt} + \lfloor l/q_k \rfloor (y_{kt} - z_{kt});\ k \in K, t \in T \tag{14}$$

where $\lfloor a \rfloor$ stands for the greatest integer not greater than a.

5. Material manufacturing and shipment constraints

– parts can only be supplied in periods scheduled for shipment, and each shipping lot is limited by its minimum and maximum allowed size $\underline{V}$ and $\overline{V}$, respectively;
– for each part type k and period t, the cumulative shipping lots in periods 1 through t cannot be greater than the initial stocks and the cumulative production of this part type in periods 1 through t:

$$v_{kt} \leqslant \overline{V} w_t;\ k \in K, t \in T \tag{15}$$

$$\sum_{k \in K} v_{kt} \leqslant \overline{V};\ t \in T \tag{16}$$

$$\sum_{k \in K} v_{kt} \geqslant \underline{V} w_t;\ t \in T \tag{17}$$

$$\sum_{\tau=1}^{t} v_{k\tau} \leqslant I_{k0}^{1} + \sum_{\tau=1}^{t} u_{k\tau};\ k \in K, t \in T \tag{18}$$

6. Material demand satisfaction constraints

– for every period t, the cumulative production of product type k in periods 1 through t cannot be greater than the initial stocks and the cumulative supplies of part type k in periods 1 through $t-1$:

$$\sum_{j \in J_k} \sum_{\tau=1}^{t} o_j x_{j\tau} \leqslant I_{k0}^{2} + \sum_{\tau=1}^{t-1} v_{k\tau};\ k \in K, t \in T \tag{19}$$

7. Inventory constraints (1)–(4)

8. Variable nonnegativity and integrality constraints

$$u_{kt} \geqslant 0;\ k \in K, t \in T \tag{20}$$

$$v_{kt} \geqslant 0;\ k \in K, t \in T \tag{21}$$

$$w_t \in \{0,1\};\ t \in T \tag{22}$$

$$x_{jt} \in \{0,1\};\ j \in J, t \in T:\ r_j \leqslant t \leqslant d_j \tag{23}$$

$$y_{kt} \geqslant 0,\ integer;\ k \in K, t \in T \tag{24}$$

$$z_{kt} \geqslant 0,\ integer;\ k \in K, t \in T \tag{25}$$

In order to satisfy non-delayed assignment constraint (7), some customer due dates are assumed to have been adjusted and delayed, while the order acceptance and due dates setting decisions are made, see Sawik (2007c).

4. Computational examples

In this section some computational examples are presented to illustrate possible applications of the proposed approach. A brief description of the manufacturing and assembly stages of the supply chain, parts, products and the customer orders is given below.

1. Planning horizon: $t_h = 30$ days, each of length $l = 2 \times 9$ hours.
2. Manufacturing:
   - $m_M = 22$ parallel production lines, each to be set up at most once per planning period, with identical start-up time $\sigma = 9000$ seconds.
   - parts:
     - $g = 10$ product-specific part types,
     - processing times q_k (in seconds) for part types $k = 1, \ldots, 10$: $q_1 = 65, q_2 = 70, q_3 = 80, q_4 = 85, q_5 = 90, q_6 = 95, q_7 = 65, q_8 = 80, q_9 = 90, q_{10} = 85$.
3. Supply:
   - at most one shipment of various parts per period,
   - the minimum and maximum allowed shipping lots: $\underline{V} = 1,000$, and $\overline{V} = 50,000$ parts,
   - transportation time constant and equal to one period for each shipment.
4. Assembly:
   - $m_A = 6$ assembly stages in series with parallel machines: $m_i = 10$ parallel machines at each stage $i = 1, 2$; $m_i = 20$ parallel machines at each stage $i = 3, 4, 5$; and $m_i = 10$ parallel machines at stage $i = 6$.
   - products:
     - $g = 10$ product types of three product groups, each to be processed on a separate group of machines (at stage 3 or 4 or 5),
     - processing times (in seconds) for product types:

product type/stage	1	2	3	4	5	6
1	20	0	120	0	0	15
2	20	0	140	0	0	15
3	10	0	160	0	0	10
4	15	5	0	120	0	15
5	15	10	0	140	0	15
6	10	5	0	160	0	10
7	15	10	0	180	0	15
8	20	5	0	0	120	15
9	15	0	0	0	140	10
10	15	0	0	0	160	10

     - $n = 805$ customer orders ranging from 5 to 9,620 products, with various requested shipping dates. The total demand is 535,000 products.
5. Initial inventory: the initial supplier and producer stocks of parts are equal to an average two-day manufacturing and assembly volumes, respectively.

Table 2. *Computational results*

Model	Var.	Bin.	Int.	Cons.	Solution values	GAP$^{(a)}$
SAMS(0.1,0.1)$^{(b)}$	3507	2307	600	3395	$H_{sum} = 2103$, $W_{sum} = 29$, $Z_{sum} = 173$	1.22%
SAMS(1,1)$^{(b)}$	3507	2307	600	3395	$H_{sum} = 2142$, $W_{sum} = 28$, $Z_{sum} = 99$	1.11%
SAMS(10,10)$^{(b)}$	3507	2307	600	3395	$H_{sum} = 2332$, $W_{sum} = 28$, $Z_{sum} = 40$	1.23%
SAMS(100,100)$^{(b)}$	3507	2307	600	3395	$H_{sum} = 3078$, $W_{sum} = 14$, $Z_{sum} = 28$	5.89%

$H_{sum} = \sum_{k \in K} \sum_{t \in T} (0.001 I^1_{kt} + 0.002 I^2_{kt} + 0.004 I^3_{kt})$ – total inventory holding cost,
$W_{sum} = \sum_{t \in T} w_t$ – total number of shipments, $Z_{sum} = \sum_{k \in K} \sum_{t \in T} z_{kt}$ – total number of start ups
$^{(a)}$ GAP% after 1800 CPU seconds on a PC Pentium IV, 1.8 GHz, RAM 1 GB/CPLEX v. 9.1
$^{(b)}$ Unit costs γ_w, γ_z in the objective function

The total system inventory for various units costs γ_w and γ_z is presented in Figure 1. The characteristics of mixed integer programs are summarised in Table 2. The size of mixed integer programs is represented by the total number of variables, Var., number of binary variables, Bin., number of integer variables, Int., and number of constraints, Cons. The last two columns of the table present the solution values and % GAP after 1,800 seconds of CPU time.

The computational experiments were performed using the AMPL programming language and the CPLEX v.9.1 solver (with the default settings) on a laptop with a Pentium IV processor running at 1.8 GHz and with 1 GB RAM.

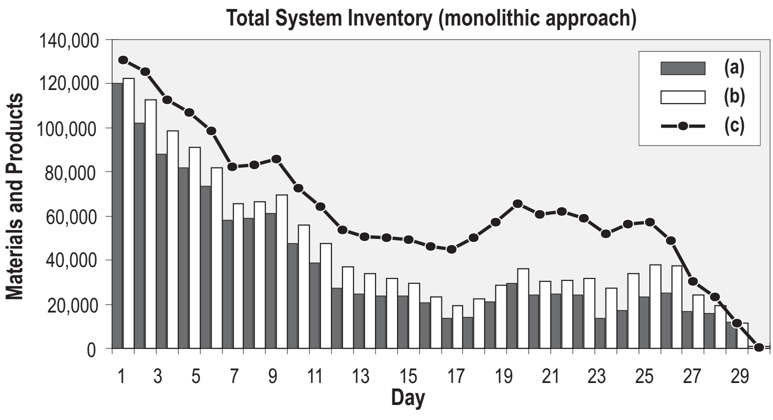

Fig. 1. *Total system inventory for* $h_1 = 0.001, h_2 = 0.002, h_3 = 0.004$*: (a)* $\gamma_w = 1, \gamma_z = 1$*, (b)* $\gamma_w = 10, \gamma_z = 10$*, (c)* $\gamma_w = 100, \gamma_z = 100$

The figure shows that lower total inventory levels are achieved for smaller unit costs γ_w and γ_z. The latter result is due to the more frequent line start ups and part supplies required to meet fluctuating demand for parts for lower start-up and shipping unit costs, which leads to reducing the total inventory.

5. Conclusion

In this paper the integration of supply, production and distribution in a customer driven supply chain is considered and a mixed integer programming formulation is proposed for the long term scheduling and coordination of parts manufacturing and supply and finished products assembly.

The computational experiments modelled after a real world customer driven supply chain in the electronics industry have indicated that the approach is capable of finding good long-term schedules for large size problems in a reasonable computation time, using commercially available software for integer programming.

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