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Special Algorithms for Assessing Market Value of Real Estates

1. Introduction

Computer systems that concern monitoring and analyzing the real estate market typically deal with information on attributes and unit transaction prices referring to previously sold estates. Such information is of historical character, hence the impossibility to accurately reconstruct the state and features of those estates as well as the conditions of the sales contracts concluded. The fundamental source for creating these bases are notarial deeds in the form of sales contracts, which only to a very limited extent contain information on the market features (attributes) of the estates that affect the transaction price.

Obligatory attributes of a real estate, concerning such properties as location, attributed rights, appropriation in the local land development plan, fittings, utilities as well as the area, can be easily established based on the formal legal documentation. Attributes of discretional (facultative) character though, such as access to public transport, environment, development degree, standard of the premises, technical state of the buildings, etc., are established, in a period of time shifted relative to the date of transaction, based on visits in the field and inside the premises, at the stage of market analysis. In many cases it is impossible for these attributes to draw up a detailed description of their properties as of the date of transaction and to assign to them values according to the established scale. The above determinants make that blank fields appear in the information bases on real estates, which indicates lack of information regarding the attribute concerned.

For real estate market analysis, always a group of representative real estates should be chosen, the structure of which reflects the structure of the features of all market transactions realized within the time period being examined.

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The base of representative real estates for the market being considered must fulfill the following conditions:

- the number of estates should be larger than 20, and for small markets, it should not be smaller than 12;
- the structure of the individual attributes of the real estates should be proportional to the structure of the corresponding attributes for all of the transactions realized in the market being examined;
- the permissible difference between the average price $\hat{c}$ of the estates within the representative base and the average price at all of the transactions of the market being considered should not exceed 20% of its value;
- the permissible difference between the price standard deviation (σ) for the estates within the representative base and the standard deviation of all the transactions of the market being considered should not exceed 30% of its value.

The characteristic features of each real estate, represented by the unit transaction price and the attributes thereof, constitute the multidimensional random variable. In the aspect of the facultative attributes of the individual real estates, these variables are only very poorly determined as the scale values of these attributes are determined by the analyst in a discretional manner.

The model of the multidimensional random variable, determined by the representative base of real estates, is characterized by precisely defined statistical parameters that have an interpretation tantamount to the indices of real estate market analysis.

The full characteristics of the multidimensional random variable, represented by the attributes and unit prices of real estates, comprise the following parameters:

- average value for the unit price or the attributes of the real estate in the marginal distribution of the multidimensional random variable;
- standard deviation for the unit price or the attributes of the real estate in the marginal distribution of the multidimensional random variable;
- total correlation coefficients for the individual attributes relative to the unit price of the estates in the marginal distribution of the multidimensional random variable;
- matrix of total correlation coefficients for the attributes of the estates and the unit prices;
- multidimensional linear correlation coefficient for the unit prices of estates relative to their attributes;
- weight coefficients β for the individual attributes relative to the unit prices of the real estates;
- probability density function of the normal (Gaussian) distribution of the multidimensional random variable.

The above mentioned parameters are the basis for constructing various statistical models of real estate valuation that lead to different methods of assessing market value of real estates.

If the base of representative real estates contains a different number of determined attribute values for different estates, then it will not be possible to use all of the parameters of the full-dimensional random variable in the market analysis. A compromise solution to such a situation would be a market analysis based exclusively on those attributes that have a full representation in the base being considered, which would imply the necessity to omit those attributes that are not determined in all of the estates being considered or to omit those estates that do not have a determined value of at least one of the attributes being considered.

For a base of representative real estates containing blank fields that inform of a lack of information on the attributes being considered, only those parameters can be used for a market analysis that refer to marginal distributions of the multidimensional random variable, i.e. the first three of the parameters listed above.

The above considerations lead to the general conclusion that for bases of representative estates with incomplete information on attributes, special methods of market analysis must be developed as well as special algorithms for assessing real estate market value. The basis for such a market analysis are appropriate similarity weights of the estate attributes the values of which should be determined, according to the Markov principle, taking into consideration known and unknown attribute values for the estates being examined.

For such weights to reflect the mutual similarity of the estate attributes as well as the likelihood of identification of the estate features, appropriate attribute scale construction should be used, according to the following principle:

5 – the most favorable feature for the potential buyer and user of the estate,
4 – very favorable feature for the potential buyer and user of the estate,
3 – favorable feature for the potential buyer and user of the estate,
2 – average feature for the potential buyer and user of the estate,
1 – unfavorable feature for the potential buyer and user of the estate,
0 – no information / assessment.

2. Real Estate Market Analysis

Preliminary analysis of the real estate market, founded on the base of representative estates with attributes scaled according to the above principles, comprises calculating the following two parameters: average value of $\hat{c}_T$ from unit transaction prices as well as standard deviation $\sigma[c_{Ti}]$ from unit transaction prices.

Based on the above parameters, it is possible to make a preliminary verification of the unit transaction prices that will be basis for eliminating from the base any states with divergent unit prices, i.e. ones satisfying the following inequality

$$\hat{c}_T - 3 \cdot \sigma[c_{Ti}] \leq c_{Ti} \leq \hat{c}_T + 3 \cdot \sigma[c_{Ti}] \tag{1}$$

If, as a result of the realization of the above criterion, estates with divergent prices are removed, then, for the reduced representative base of real estates, the average value of the unit transaction price and its standard deviation should be calculated again. These parameters are related to different transaction times, so they cannot be directly used for valuating the estates, they can only serve the purpose of a general characteristics of the unit transaction prices within the verified representative base of real estates.

Real estate market analysis should be carried out in the following steps:

1. Defining the mutual similarity weights of the attributes relative to their central measure

These weights should account for the number of attributes being considered and their values, taking also into account any lack of information on some of the attributes, i.e. '0' values. Using the principles of constructing weights, provided by Markov, similarity weights of estate attributes shall be defined according to the following formula

$$p_i = \frac{s}{a + \sum_{j=1}^{s} (a_{ij} - \hat{a}_j)^2} \tag{2}$$

where:

s – the number of all attributes being considered within the base of real estates,

$i = 1, 2, \ldots, n$ – estate number in the base being considered,

a_{ij} – the value, according to a definite scale, of the j-th attribute for the i-th estate, also '0' values,

$\hat{a}_j$ – average value of the j-th attribute in the base being considered, calculated with omission of '0' values.

Weights defined according to the formula (2), take into account the number of the attributes being considered in different estate bases, similarity of estate attributes relative to their central measure, determined on quantified attributes; they also present the likelihood degree of the description of all the features of the estates being considered. The values of these weights may be contained in the interval from to a value near.

The calculated similarity weights of the attributes of the estates constitute the basis for analyzing the state of the market over time and for adjusting transaction prices as of the date of the market analysis, as well as for determining the influence of the individual attributes on the variation of the adjusted prices.

2. Determining variation trend of the unit transaction prices over time

In real estate market analyses, unit prices of the estates selected to be in the base should be adjusted (transformed) as of the date of valuation. If this adjustment is to be based on market tendencies, then the variation trend (formula) concerning real estate prices over the definite time must be determined. The basis for the trend estimation should be identical or very similar estates whose transactions were realized over a period of at least a dozen months of the time interval being considered. Due to the fact that in different time-shifted transactions typically there are estates with different attribute values, the price variation trend should be estimated with the use of functions containing a time variable expressing transaction shift in months, but taking into account the similarity weights of the estate attributes.

If the market analysis employs a model in the form of an exponential function in which prices should be adjusted proportionally do their values in the subsequent months (t), then the function relation between the unit transaction price and the time of transaction assumes the form

$$c_T = b_0 \cdot b^t \tag{3}$$

where:

c_T – unit transaction price of an estate,
b_0 – parameter describing the forecast prices of the estate as of the date of the first transaction (in the zero month),
b – parameter determining price variability coefficient for one month,
t – number of months that elapsed from the date of the transaction being considered relative to the date of the first transaction in the base.

Parameters in model (3) should be estimated using the least squares method, but taking into account the similarity weights of the attributes, calculated according to formula (2).

3. Adjustment of unit transaction prices as of the date of the analysis (valuation)

On the basis of the definite values of the parameters in model (3), adjustment of the unit transaction prices must be made as of the date of the analysis (valuation), according to the following formula

$$c_{ki} = c_{Ti} + b_0 \cdot (b^{t_w} - b^{t_i}) \tag{4}$$

where:

c_{ki} – unit transaction price of the i-th estate adjusted as of the date of the analysis,

c_{Ti} – unit transaction price of the i-th estate,

b_0 – estimated parameter describing the forecast price of the estate as of the date of the first transaction (zero month),

b – estimated parameter determining price variability coefficient for one month,

t_w – number of months that elapsed from the date of the analysis (valuation) relative to the date of the first transaction in the base,

t_i – number of months that elapsed from the date of the i-th transaction relative to the date of the first transaction in the base.

4. Determining the impact of the individual attributes on variability of unit transaction prices adjusted as of the date of the analysis

For real estate bases containing attributes the values of which have not been determined, i.e. with the '0' symbols, it is not possible to determine a matrix of correlation coefficients or to carry out a complete statistical market analysis. In such bases it is possible to determine, for each attribute independently, complete correlation coefficients, with adjusted prices, but taking into account previously determined similarity weights for attributes of the individual estates.

For each attribute of the representative base of real estates, only these estates should be selected that have definite attribute values, and for them the complete correlation coefficient relative to the adjusted unit prices of these estates should be calculated.

If pairs (a_i, c_{ki}) of the values of the selected attribute and the adjusted unit price, for $i = 1, 2, \ldots, n$ estates, represent two-dimensional random variable, then the estimator of the complete correlation coefficient will be the value $\bar{r}$, calculated according to the following formula

$$\bar{r} = \frac{\sum_{i=1}^{n} (a_i - \hat{a})(c_{ki} - \hat{c}_k) \cdot p_i}{\sqrt{\sum_{i=1}^{n} (a_i - \hat{a})^2 \cdot p_i} \cdot \sqrt{\sum_{i=1}^{n} (c_{ki} - \hat{c}_k)^2 \cdot p_i}} \tag{5}$$

at the same time:

$$\hat{a} = \frac{\sum_{i=1}^{n} a_i \cdot p_i}{\sum_{i=1}^{n} p_i}, \quad \hat{c}_k = \frac{\sum_{i=1}^{n} c_{ki} \cdot p_i}{\sum_{i=1}^{n} p_i} \tag{6}$$

denote average value of the attribute being considered and of the adjusted unit price, calculated as weighted average, for all the estates in the base being analyzed, but with definite values of the attributes.

The estimator of the complete correlation coefficient constitutes the strength of linear codependence between the unit price and the particular attribute in the base being analyzed.

Of special significance is the value of the coefficient ($\bar{r}$) relative to its confidence interval resulting from inaccuracy of statistical estimations of the parameters of the base concerned. In this scope, a criterion must be formulated that will make it possible to eliminate from further market analysis the coefficient ($\bar{r}$) with insignificant values, which will be the subject of subsequent publications.

Of special significance is the value ($\bar{r}^2$), which can constitute the explanation measure of variable c by the model of linear regression relative to the variable a being considered. Using variance analysis for two-dimensional random variables a and c, it can be proved that ($\bar{r}^2$) determines what share of the variability of unit prices in the real estate base is explained independently by the attribute being considered.

Standardized weight shares (k_j) of the particular attributes in explaining the variance of the adjusted unit prices can be most conveniently determined on the basis of standardization of value ($\bar{r}_j^2$), i.e. according to the formula

$$k_j = \frac{\bar{r}_j^2}{\sum_{j=a}^{u} \bar{r}_j^2} \tag{7}$$

with the proviso that taken into account are only those values r_j that had been positively verified concerning their significance.

Weight shares of the particular attributes, determined according to the formula (7), can make the base for developing special algorithms for assessing market values of any real estates.

3. Algorithm for Assessing Real Estate Market Value According to a Two-Dimensional Regression Model

The basis for using the two-dimensional regression model is a base of similar real estates, which in some cases can constitute a base of representative real estates, considered in the market analysis. Formulation of the algorithm will be accomplished using similarity weights (p_i) for all the particular attributes of the es-

tates in the base as well as standardized weight shares (k_j) of the individual attributes.

The algorithm of assessing real estate market value requires carrying out the following steps:

1. Describing the estate being valuated using previously scaled attributes $\{\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n\}$.
2. Calculating the value of average adjusted unit prices of the estates in the base to be compared.
3. Calculating standard deviation of unit prices of the estates in the base to be compared.
4. Calculating average value of the individual attributes of the estates in the base to be compared.
5. Calculating standard deviation of the values of the individual attributes of the estates in the base to be compared, with omission of '0'.
6. Selecting complete correlation coefficients ($\bar{r}_j$) between the j-th attribute and the estate price from the analysis of the representative base of real estates that fulfill the criterion of significance of their values.
7. Selecting standardized weight shares (k_j) of the individual attributes from the analysis of the representative base of real estates.
8. Forecasting the value of market unit price of the estate according to the following formula

$$w = \hat{c}_k + \sum_{j=1}^{s} \frac{\bar{r}_j \cdot \sigma[c_{ki}] \cdot k_j}{\sigma[a_j]} (\bar{a}_j - \hat{a}_j) \tag{8}$$

with the proviso that all quantities appearing in this formula are calculated taking into account similarity weights of the attributes, i.e.:

$\hat{c}_k$ – average value of adjusted unit prices,
$\bar{r}_j$ – estimator of the complete correlation coefficient for the j-th attribute the value of which is calculated according to the broad base of real estates,
$\sigma[c_{ki}]$ – standard deviation for adjusted unit prices,
k_j – standardized weight share of the j-th attribute the value of which is calculated according to the broad base of real estates,
$\sigma[a_j]$ – standard deviation for the j-th attribute,
$\bar{a}_j$ – the value of the j-th attribute of the estate being valuated,
$\hat{a}_j$ – average value of the j-th attribute,

9. Calculating the full market value of the real estate being evaluated as the product of its area and the market unit price assessed according to formula (8), i.e.

$$W = S \cdot w \tag{9}$$

An important issue is assessing congruity of the valuation model with the base of real estates to be compared as well as determining standard deviation for the market value of the estate being valuated. These issues will be discussed in subsequent academic publications.

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